LCR Circuit

Objectives

After going through this module learner will be able to

- Explain the special characteristics of a series LCR circuit
- State the mathematical expression for the voltage in a series LCR circuit and use it to derive the relationship between current, impedance and phase
- Recognize that a series LCR circuit can show the phenomenon of resonance and obtain the expression for the resonant frequency
- Understand the behavior of LCR circuit under resonance conditions
- Describe in detail the concept of 'sharpness of resonance' and 'quality factor' for a series LCR circuit and show how they can be increased
- Obtain the expression for the average power, over a cycle, for an AC circuit and define the term 'power factor'
- Appreciate 'wattles current'

Content Outline

- Unit Syllabus
- Module Wise Distribution of Unit Syllabus
- What You must Know Before You Start
- Introduction
- LCR Series Circuit
- Phasor Diagram Solution for LCR Circuit - Calculation of Z and Phase Angle ϕ between Current and Voltage
- Resonance in LCR circuit
- Quality Factor Q

Unit Syllabus

Unit IV: Electromagnetic Induction and Alternating Currents

Chapter-6: Electromagnetic Induction

Electromagnetic induction; Faraday's laws, induced emf and current; Lenz's Law, Eddy currents; Self and mutual induction.

Chapter-7: Alternating Current

Alternating currents, peak and rms value of alternating current/voltage; reactance and impedance; LC oscillations (qualitative treatment only), LCR series circuit, resonance; power in AC circuits, wattless current; AC generator and transformer.

Module Wise Distribution of Unit Syllabus

The above unit is divided into 9 modules for better understanding.

Module 1	● Electromagnetic induction ● Faraday's laws, induced emf and current; ● Change of flux ● Rate of change of flux
Module 2	● Lenz's Law, ● Conservation of energy ● Motional emf
Module 3	● Eddy currents. ● Self induction ● Mutual induction. ● Unit ● Numerical
Module 4	● AC generator ● Alternating currents, ● Representing ac ● Formula ● Graph ● Phasor ● Frequency of ac and what does it depend upon ● peak and rms value of alternating current/voltage;
Module 5	● ac circuits ● Components in ac circuits ● Comparison of circuit component in ac circuit with that if used in dc circuit ● Reactance mathematically ● Pure R ● Pure L

	• Pure C • Phasor, graphs for each
Module 6	• AC circuits with RL, RC and LC components • Using phasor diagram to understand current and voltage phase differences • Impedance; LC oscillations (qualitative treatment only), • Resonance
Module 7	• Alternating voltage applied to series LCR circuit • Impedance in LCR circuit • Phasor diagram • Resonance • Quality Factor • Power in ac circuit • Power factor • Wattles current
Module 8	• Transformer
Module 9	• Advantages of ac over dc • Distribution of electricity to your home

Module 7

What You Must Know

Let us remember the words we have been using in our study of this physics course:

- **Electromagnetic Induction:** The phenomenon in which electric current can be generated by varying magnetic fields is called electromagnetic induction (EMI).
- **Magnetic Flux:** Just like electric flux, magnetic flux $Ø_B$ through any surface of area A held perpendicularly in magnetic field B is given by the total number of magnetic lines of force crossing the area. Mathematically, it is equal to the dot product of B and A.

  $\Phi_B = B.A = BA\cos\theta$, where θ is the angle between B and A
- **Induced emf and Induced Current:** The emf developed in a loop when the magnetic flux linked with it changes with time is called induced emf when the conductor is in the form of a closed loop, the current induced in the loop is called an induced current.

- **Weber:** One weber is defined as the amount of magnetic flux, through an area of $1m^2$ held normal to a uniform magnetic field of one tesla. The SI unit of magnetic flux is weber (Wb) or tesla metre squared (Tm^2).
- **Faraday's Laws of Electromagnetic Induction:**
  - **First Law:** It states that whenever the amount of magnetic flux linked with the coil changes with time, an emf is induced in the coil. The induced emf lasts in the coil only as long as the change in the magnetic flux continues.
  - **Second Law:** It states that the magnitude of the emf induced in the coil is directly proportional to the time rate of change of the magnetic flux linked with the coil.
- **Lenz's Law:** The law states that the direction of induced emf is always such that it opposes the change in magnetic flux responsible for its production.
- **Induced emf by Changing the Orientation of Coil and Magnetic Field:** When the coil rotates in a magnetic field the angle θ changes and magnetic flux linked with the coil changes and this induces the emf. This is the basis of ac generators.
- **Electric Current:** An electric current equals the rate of flow of electric charge. In electric circuits this charge is often carried by moving electrons in a wire. It can also be carried by ions in an electrolyte, or by both ions and electrons such as in plasma.
- **Voltage:** The difference in electric potential energy between two points per unit electric charge, in an electric circuit.
- **Ohm's Law:** Electric current through a conductor is directly proportional to the potential difference across the conductor provided the temperature and physical conditions of the conductor remain the same.
- **Ohmic Conductors:** Conductors that follow ohm's law for a reasonable range of physical conditions like conductor wires, conductor plates, strips.
- **Non-Ohmic Conductors:** Conductors that do not follow Ohm's law e.g electrolytes, semiconductors.
- **Phasors:** In Physics and Engineering, a phasor, is a complex number representing a sinusoidal function whose amplitude (A), angular frequency (ω), and initial phase (θ) are time-invariant. Basically, Phasors are rotating vectors.
- **Alternating Voltage:** The electric mains supply in our homes and workplaces is a voltage that varies like a function with time. Output from an ac generator.
- **Alternating Current:** Current in a circuit driven by ac voltage is called ac current.

- Alternating currents and voltages have **frequency f** and **angular frequency 2πf** associated with it.
- Two currents, two voltages or currents and voltages may have a phase relation between them. This arises due to electromagnetic induction, self induction or time rate associated with charging and discharging of capacitors.
- Alternating currents and voltages have **instantaneous value** given by:

  $i = i_0 sin(\omega t + \varphi)$

  $V = V_0 sin(\omega t + \varphi)$

  Φ is the initial phase of the sinusoidal current or voltage.
- Alternating currents and voltages have **peak value I_0 and V_0.**
- Alternating currents and voltages have **average value over half cycle is given by:**

$$V_{avg(T/2)} = \frac{2V_0}{\pi} \cong 0.636\, V_0$$

- Alternating currents and voltages have **root mean square values:**

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

- **Self Inductance of a Coil L:** An electric current can be induced in a coil by flux changes produced by the changing current in itself:

$$L = \mu_0 n^2 A l$$

  Where nl = N total number of turns of the coil, A area of the face of the coil, μ_0 is the permeability of free space. The SI unit is henry.

  Self inductance is also **called back emf**. It depends upon the geometry of the coil and permeability of the medium inside the coil.
- **Energy Required to build up Current I** in a coil of inductance $L = \frac{1}{2} L I^2$.
- **Capacitor:** A system of two conductors separated by insulator parallel plate capacitors, spherical capacitors are used in circuits. Capacity of parallel plate capacitor is given by

  $c = \frac{\mu_0 A}{d}$. An area of the plate, d separation between the plates, μ_0 is the permeability of free space.

  Capacitors block dc but ac continues as charging and discharging of the capacitor maintains a continuous flow of current.
- **Capacitance:** $C = \frac{Q}{V}$ S I unit farad.

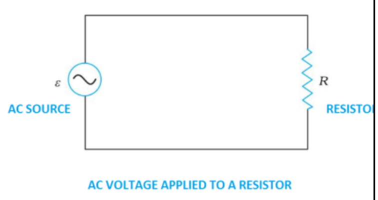
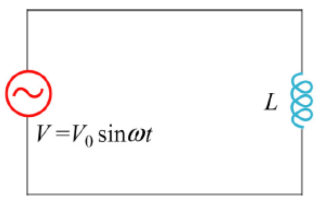
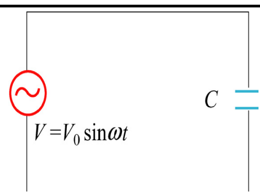
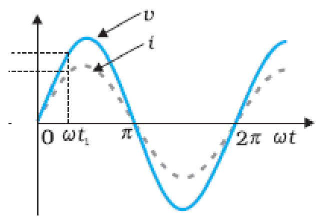
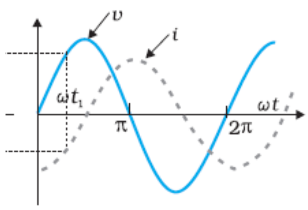
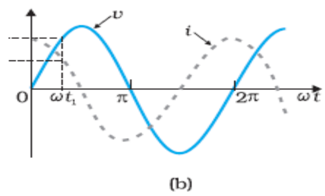
- **Dielectric Constant of a Material K:** It is the factor by which the capacitance increases from its vacuum value when the dielectric (material) is inserted fully between the plates of a capacitor.
- **Combination of Capacitors:** Capacitors may be combined in ways to obtain a value of effective capacitance.
- **Series Combination:** capacitances are said to be in series if the effective combined capacitance C is given by:

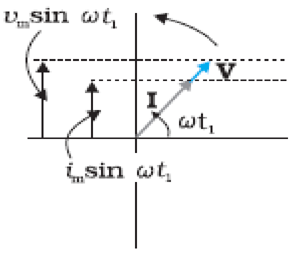
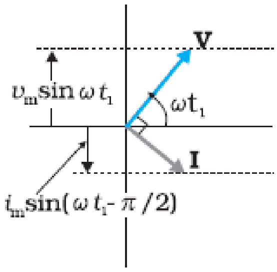
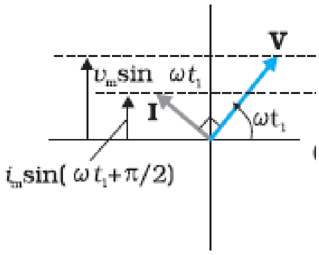
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots\ldots + \frac{1}{C_n}$$

- **Parallel Combination of Capacitors:** capacitances are said to be in series if the effective combined capacitance C is given by:

$$C = C_1 + C_2 + \ldots\ldots + C_n$$

- **Choke Coil:** In electronics, a choke is an inductor used to block higher-frequency alternating current (AC) in an electrical circuit, while passing lower-frequency currents or direct current (DC).

	Pure resistance circuit	Pure inductive circuit	Pure capacitive circuit
Circuit diagram			
Input voltage	V=$V_0 sin\omega t$	V=$V_0 sin\omega t$	V=$V_0 sin\omega t$
Current	I=$\frac{V_0}{R} sin\omega t$ I=$I_0 sin\omega t$	$I = I_0 sin(\omega t - \frac{\pi}{2})$	$I = I_0 sin(\omega t + \frac{\pi}{2})$
reactance	**R**	**X_L=2πfL**	**X_C=1/2πfC**
Current voltage graph			

	Current and voltage are in the same phase	**Voltage leads the current by π/2**	**Voltage leads the current leads voltage by π/2**
Current voltage phasor			

- **Electric Generator:** In electricity generation, a generator is a device that converts mechanical energy to electrical energy for use in an external circuit.
- **Impedance:** Electrical impedance is the measure of the opposition that a circuit presents to a current when a voltage is applied. In quantitative terms, it is the complex ratio of the voltage to the current in an alternating current (AC) circuit.
- **Reactance:** In electrical and electronic systems, reactance is the opposition of a circuit element to a change in current or voltage, due to that element's inductance or capacitance.
- **Phasors:** In physics and engineering a phasor, is a complex number representing a sinusoidal function whose amplitude (A), angular frequency (ω), and initial phase (θ) are time-invariant. Basically, Phasors are rotating vectors.

Introduction

In the previous modules we learnt that the current and voltage of a resistance inductor (RL) circuit would have a phase difference. This was also true for resistance capacitor circuits, and LC circuits. These were due to self inductance and charging discharging of inductor and capacitor in an alternating current circuit .You learnt the condition of resonance in an LC circuit when the reactance offered by inductor and capacitor were equal .

This was not the case with direct current circuits.

Now we will study LCR circuits. We will limit our study to series arrangement.

LCR Series Circuit

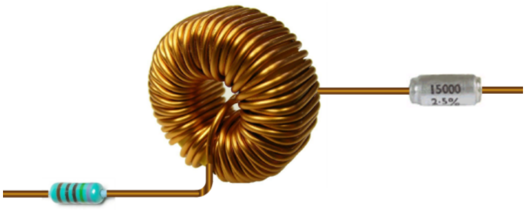
An LCR circuit is an alternating current circuit consisting of

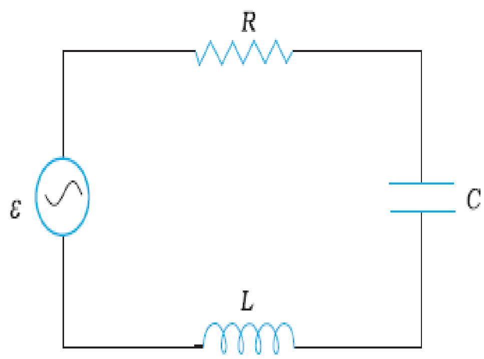
- **a resistor of resistance (R),**

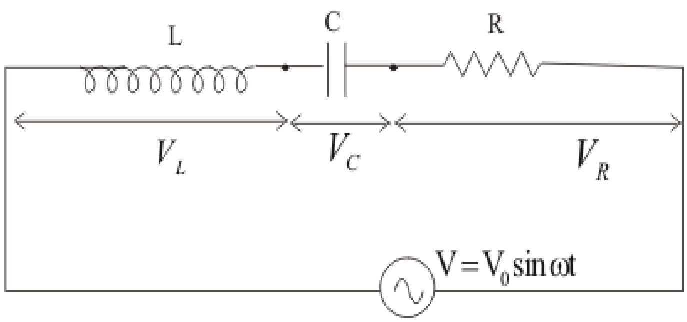
- **a coil or an inductor of inductance (L), and**
- **a capacitor of capacitance (C), connected in series or parallel .**

But we will only study R, L, C in series.

The name of the circuit is derived from the letters that are used to denote the constituent components of this circuit, where the sequence of the components may vary from LCR. So LCR, RLC, CLR, RCL etc mean the same.







Or

The figure shows a series LCR circuit connected to an ac source ε.

As usual, we take the voltage of the source to be:

$V = V_0 sin\omega t$

Same amount of current will flow in all the three circuit components and vector sum of potential drop across each component would be equal to the applied voltage as per Kirchhoff's rule.

If I be the amount of current in the circuit at any time and V_L, V_C and V_R the potential drop across L,C and R respectively then

$V_R = I R$ ⇒ Voltage is in phase with I,

$V_L = I \omega L$ ⇒ Voltage is leading current by 90^0.

$V_C = I\, 1/\omega C$ ⇒ Voltage is lagging behind current by 90^0

Since, V_L is ahead of I by 90^0 and V_C is behind by 90^0.

So that phase difference between V_L and V_C is 180^0 and they are in direct opposition to each other.

If q is the charge on the capacitor and I the current, at time t, we have, from Kirchhoff's loop

$$L\frac{dI}{dt} + IR + \frac{q}{C} = V$$

We want to determine the instantaneous current I and its phase relationship to the applied alternating voltage V.

We can solve this problem by two methods.

- **First method - we use the technique of phasors and**
- **Second method, -we solve** $L\frac{dI}{dt} + IR + \frac{q}{C} = V$ **Analytically to determine the time dependence of I.**

  In this course, we will only learn the phasor diagram method.

Phasor Diagram Solution for LCR Circuit

Calculation of Z

Let us write the equations of current and voltage for all elements of the circuit separately

For resistance

$I = I_0 sin\omega t$

$V = V_0 sin\omega t$

For inductor or coil only

$I = I_0 sin\, \omega t$

$V = V_0 sin(\omega t + \frac{\pi}{2})$

For capacitor only

$I = I_0 sin\, \omega t$

$V = V_0 sin(\omega t - \frac{\pi}{2})$

We see that the resistor, inductor and capacitor are in series.

Therefore, **The ac current in each element is the same at any time, having the same amplitude and phase.**

Let it be

$I = I_0 sin(\omega t + \phi)$

where ϕ is the phase difference between the voltage across the source and the current in the circuit.

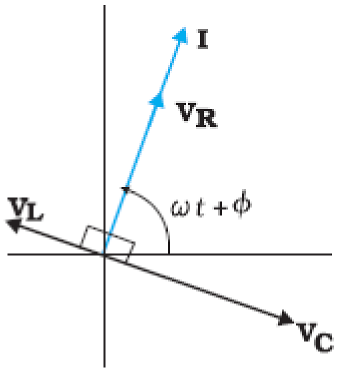
On the basis of what we have learnt in the previous modules, we shall construct a phasor diagram for the present case.

Let I be the phasor representing the current in the circuit as given by $I = I_0 sin(\omega t + \phi)$

Further, let V_L, V_R, V_C, and V represent the voltage across the inductor, resistor, capacitor and the source, respectively.

From the previous module, we know that V_R is parallel to I, V_C is π/2 behind I and V_L is π/2 ahead of I.

V_L, V_R, V_C and I can be represented as



The phasors show appropriate phase relation.The length of the phasors or amplitude of V_R , V_L , V_C are

$V_{RO} = I_0 R$

$V_{C0} = I_0 X_C$

$V_{L0} = I_0 X_L$

The voltage equation for the LCR circuit is

$V_L + V_C + V_R = V$

This relation is represented in the phasor. Since, V_C and V_L are always along the same line and in opposite directions, they can be combined into a single phasor (V_C + V_L) which has a magnitude: $\lfloor V_{C0} - V_{L0} \rfloor$

Since V is represented as the hypotenuse of a right-triangle whose sides are V_R and (V_C + V_L), the Pythagorean theorem gives:

$$V_0^2 = V_R^2 + \left(V_{C0} - V_{L0}\right)^2$$

$$V_0^2 = I_0^2 \lfloor R^2 + \left(X_C - X_L\right)^2 \rfloor$$

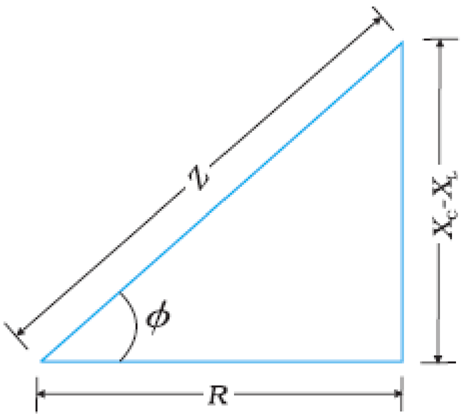
Or

$$I_0 = \frac{V_0}{\sqrt{R^2 + \left(X_C - X_L\right)^2}}$$

And for LCR circuit impedance Z will be given by

$$Z = \sqrt{R^2 + \left(X_C - X_L\right)^2}$$

Since phasor I is always parallel to phasor V_R, the phase angle ϕ is the angle between V_R and V and can be determined from



- **This is called an Impedance *diagram* which is a right-triangle with *Z* as its hypotenuse.**

  $$tan\,\phi = \frac{V_{C0} - V_{L0}}{V_{R0}}$$

  or

  $$tan\,\phi = \frac{X_C - X_L}{R}$$

- **If $X_C > X_L$, ϕ is positive and the circuit is predominantly capacitive. Consequently, the current in the circuit leads to the source voltage.**

- **$X_C < X_L$, ϕ is negative and the circuit is predominantly inductive. Consequently, the current in the circuit lags the source voltage.**

A quick way to remember whether current/voltage will lead is to remember the word CIVIL

See the spelling carefully

C capacitive

I current

V voltage

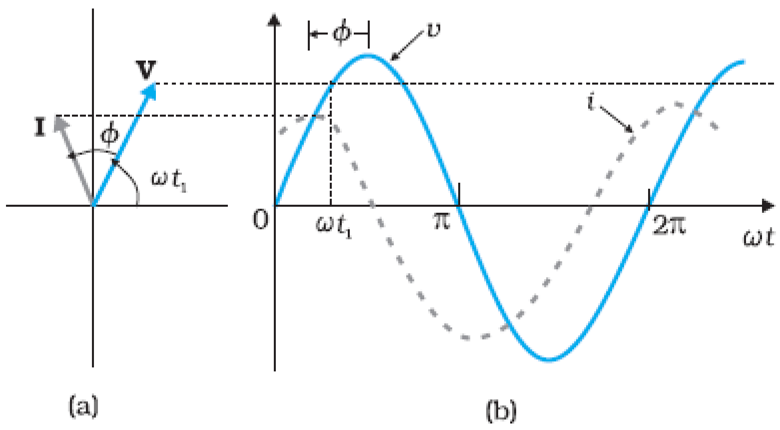
L inductive

In capacitive circuit $X_C > X_L$ CURRENT LEADS THE VOLTAGE CIVIL

Alphabet I comes before V after C (capacitor)

In inductive circuit $X_C < X_L$ CURRENT LAGS THE VOLTAGE CIVIL

Alphabet I comes after V before L (inductor)



The figure (a) below shows the phasor diagram of V and I

The Figure (b) shows the variation of V and I with ωt for $X_C > X_L$

Thus, we have obtained the amplitude and phase of current for an *LCR* series circuit using the technique of phasors. But this method of analyzing ac circuits suffers from certain disadvantages.

First, the phasor diagram says nothing about the initial condition. One can take any arbitrary value of t (say, t_1, as done throughout this module) and draw different phasors which show the relative angle between different phasors.

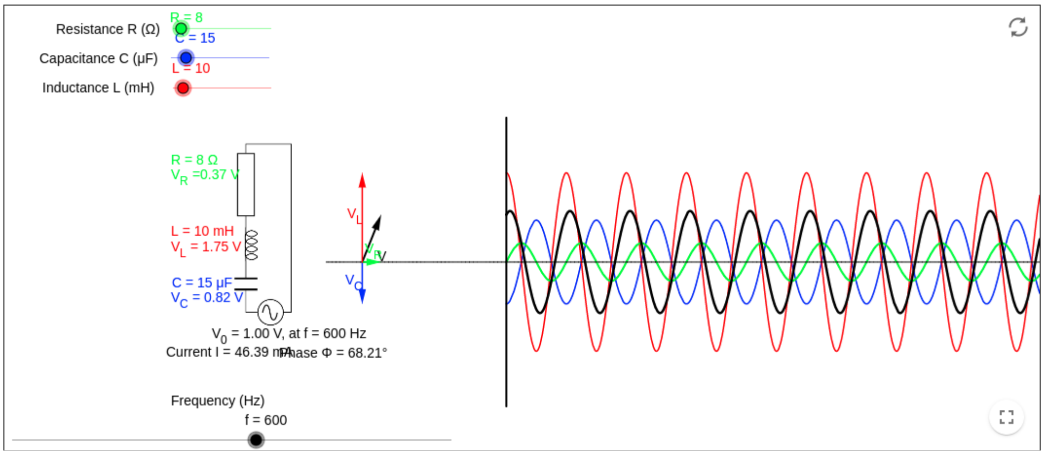
Think About This

- **How would the phasor diagrams for an inductance dominated LCR circuit differ from a Capacitance dominated LCR circuit?**
- **When would the phase difference between current and voltage be 0?**
- **When would the phase difference between current and voltage be $\frac{\pi}{2}$?**
- **Can the phase difference be greater than π?**

The solution obtained is called the ***steady-state solution***. This is not a general solution.

We will not consider the analytical method which allows us to work even in a transient situation.

Using GeoGebra App to see the phasors and variation of voltage and current with time.



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Resonance

In an LCR circuit the circuit will behave inductive or capacitive.

From this equation three cases arises:

$$Z = \sqrt{R^2 + \left(X_C - X_L\right)^2}$$

$$X_C = 1/\omega C$$

$$X_L = \omega L$$

There will be three cases

i. **When $\omega L > 1/\omega C$, then $\tan\phi$ is positive i.e. ϕ is positive and voltage leads the current I.**

ii. **When $\omega L < 1/\omega C$, then $\tan\phi$ is negative i.e. ϕ is negative and voltage lags behind the current I**

iii. **When $\omega L = 1/\omega C$,then $\tan\phi$ is zero i.e. ϕ is zero and voltage and current are in phase.**

Again considering case (iii), when $\omega L = 1/\omega C$, we have

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$Z = R$$

which is the minimum value Z can have.

The case where $X_L = X_C$, the circuit is said to be in electric resonance where the impedance is purely resistive and minimum and currents have its maximum value.

Hence at resonance

$$\omega L = \frac{1}{\omega C}$$

or

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

But $\omega = 2\pi f$ where f is the frequency of applied voltage.

Therefore, $f_0 = \frac{1}{2\pi\sqrt{LC}}$

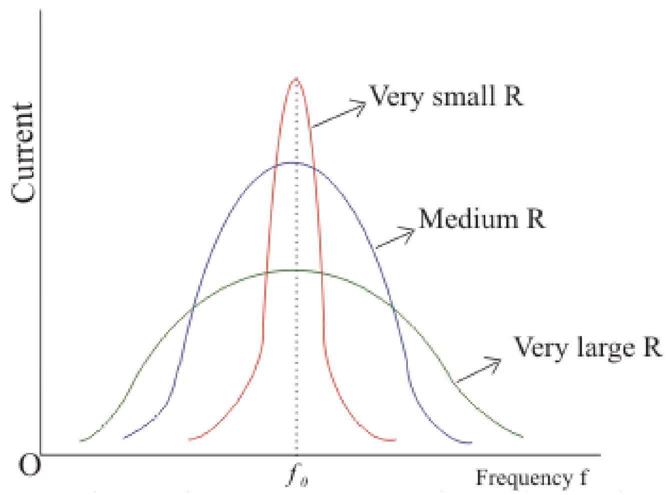
This **frequency is called resonant frequency f_0** of the circuit and **peak current** in this case is:

$I_0 = V_0/R$, and **reactance** is zero.

Interesting because the circuit impedance Z = R

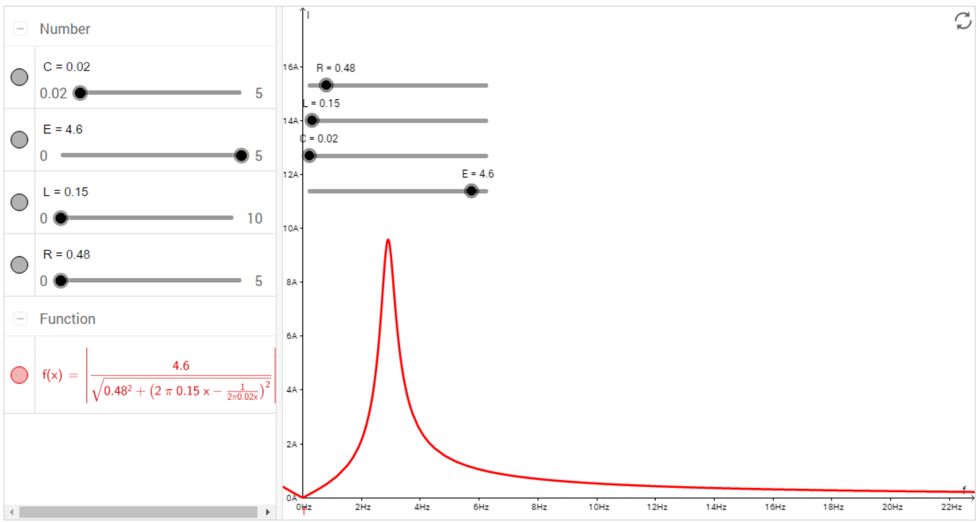
We will now define **Resonance Curves** which show the variation in circuit current (peak current i_0) with change in frequency of the applied voltage.

Figure below shows the shape of **resonance curve for various values of resistance R**



Frequencies at different R

Resonance



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For small value of R, the resonance is sharp which means that if applied frequency is lesser to resonant frequency f_0, the current is high Otherwise

For large values of R, the curve is broad sided which means that those is limited change in current for resonance and non -resonance conditions

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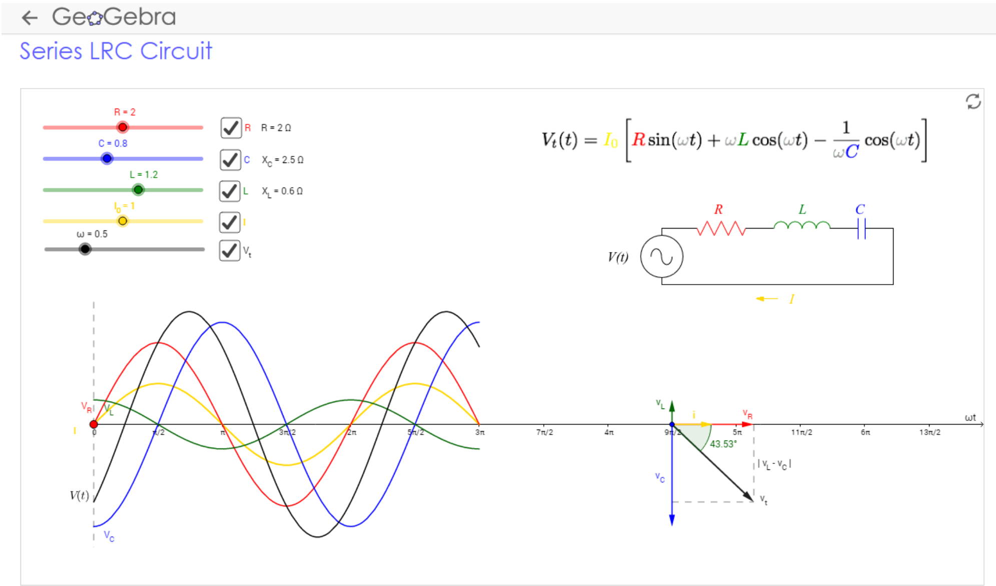


Here we have learnt an interesting characteristic of the series **LCR** circuit is the phenomenon of resonance.

The phenomenon of resonance is common among systems that have a tendency to oscillate at a particular frequency. This frequency is called the system's Natural frequency.

If such a system is driven by an energy source at a frequency that is near the natural frequency, the amplitude of oscillation is found to be large.

Resonant circuits have a variety of applications, for example, in the tuning mechanism of a radio or a TV set. The antenna of a radio accepts signals from many broadcasting stations. The signals picked up in the antenna acts as a source in the tuning circuit of the radio, so the circuit can be driven at many frequencies.

But to hear one particular radio station, we tune the radio. In tuning, we vary the capacitance of a capacitor in the tuning circuit such that the resonant frequency of the circuit becomes nearly equal to the frequency of the radio signal received. When this happens, the amplitude of the current with the frequency of the signal of the particular radio station in the circuit is maximum.

It is important to note that resonance phenomenon is exhibited by a circuit only if both L and C are present in the circuit. Only then do the voltages across L and C cancel each other (both being out of phase) and the current amplitude is V_0/R, the total source voltage appearing across R. This means that we cannot have resonance in a RL or RC circuit.

Study the following examples:

Example:

Determine the impedance of a series LCR-circuit if the reactance of C and L are 250 Ω and 220 Ω respectively and R is 40 Ω.

Solution:

Here $X_C = 250\ \Omega$

$X_L = 220\ \Omega$

$R = 40\ \Omega$

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{40^2 + (220 - 250)^2}$

$= \sqrt{1600 + 900} = \sqrt{2500} = 50\ \Omega$

Example:

A resistor of 50 Ω, an inductor of (20/π)H and a capacitor of (5/π)μF are connected in series to a voltage source 230 V, 50 Hz. Find the impedance of the circuit.

Solution:

Here $R = 50\ \Omega$

$L = (20/\pi)\ H$

$C = (5/\pi)\ \mu F = 5/\pi \times 10^{-6}\ F$

$V = 230\ V$

$f = 50\ Hz$

$X_L = 2\pi f L = 2 \times \pi \times 50 \times \frac{20}{\pi} = 2000\ \Omega$

$X_C = \frac{1}{C \times 2\pi f} = \frac{1}{\frac{5}{\pi} \times 10^{-6} \times 2 \times \pi \times 50} = 2000\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{50^2 + (2000 - 2000)^2} = \sqrt{2500} = 50\ \Omega$

Example:

A 0.3 H inductor, 60μF capacitor and a 50 Ω resistors are connected in series with a 120 V, 60 Hz supply. Calculate:

(i) Impedance of the circuit

(ii) Current flowing in the circuit

(iii) Predict whether the circuit is capacitive or inductive

(iv) Determine the phase angle by which current/voltage leads voltage or current

Solution:

Here L = 0.3 H

C = 60 μF = 60 × 10^{-6} F

R = 50 Ω

V = 120 V

f = 60 Hz

(i) Inductive reactance, $X_L = 2\pi f L = 2 \times 3.14 \times 60 \times 0.3 = 113.04\ \Omega$

Capacitive reactance, $X_C = \frac{1}{C \times 2\pi f} = \frac{1}{60 \times 10^{-6} \times 2 \times 3.14 \times 60} = 44.23\ \Omega$

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{50^2 + (113.04 - 44.23)^2} = \sqrt{50^2 + (68.81)^2}$

$= \sqrt{2500 + 4734.8} = \sqrt{7234.8} \simeq 85\ \Omega$

(ii) Current in the circuit is:

$I = \frac{V}{Z} = \frac{120}{85} = 1.41$ A

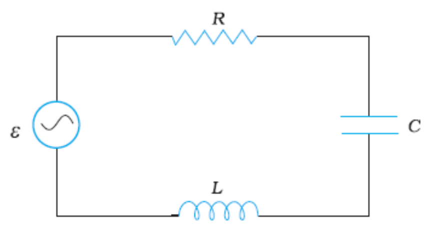
(iii) $X_L > X_C$ the circuit is inductive , using CIVIL voltage leads the current.

(iv) Phase angle $\phi = tan^{-1}(\frac{X_L - X_C}{R})$

$= tan^{-1}(\frac{113.04 - 44.23}{50}) = tan^{-1}(\frac{68.81}{50}) = tan^{-1}(1.37) = 53.87\ degrees$

Example:

Figure shows series LCR circuit with L = 5.0. H, C = 80 μF, R = 40 Ω connected to a variable frequency 240 V source.



Calculate:

(i) The angular frequency of the source which drives the circuit at resonance.

(ii) The current at the resonating frequency.

(iii)The rms potential drops across the capacitor at resonance.

Solution:

Here, V_{rms} = 240 V

L = 5.0.H

C = 80 μF = 80 × 10^{-6} F

R = 40 Ω

(i) $\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = 50\, rad\, s^{-1}$

(ii) At resonance, Z = R = 40 Ω

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{240}{40} = 6\, A$$

(iii) rms potential drop across the capacitor at resonance =

$$I_{rms} \times \frac{1}{\omega_r C} = 6 \times \frac{1}{50 \times 80 \times 10^{-6}} = 1500 V$$

Example:

A resistor of 200 ohms and a capacitor of 15.0 μF are connected in series to a 220 V, 50 Hz ac source.

a. Calculate the current in the circuit;
b. Calculate the voltage (rms) across the resistor and the capacitor.
c. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox

Solution:

R 200 ohms, *C* = 15.0 X10^{-6} F

V = 220 V, f = 50Hz

(a) In order to calculate the current, we need the impedance of the

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(200)^2 + \frac{1}{\left(2 \times 3.14 \times 50 \times 15 \times 10^{-6}\right)^2}}$$

$$= \sqrt{(200)^2 + (212)^2} = 291.5\, ohms$$

Current in the circuit is $\frac{V}{Z} = \frac{220}{291.5} = 0.755 A$

(b) Since the current is the same throughout the circuit, we have

V_R = I R = (0.755 A)(200) =151V

V_C = I X_C = (0.755 A)(212.3)=160.3

The algebraic sum of the two voltages,

V_R and V_C is 311.3 V which is more than the source voltage of 220 V. How to resolve this paradox?

As you have learnt in the text, the two voltages are not in the same phase. Therefore, **they cannot be added like ordinary numbers.**

The two voltages are out of phase by ninety degrees. Therefore, the total of these voltages must be obtained using the Pythagorean theorem

$$V_{R+C} = \sqrt{V_R^2 + V_C^2} = 220V$$

Thus, if the phase difference between two voltages is properly taken into account, the total voltage across the resistor and the capacitor is equal to the voltage of the source.

Example:

Metal detectors work on the principle of resonance in LCR circuits. Explain.

Explanation: Imagine the metal detector looking out for weapons, installed at public places for security reasons (at airports, metro stations, railway stations, mela entry points) when we walk through the metal detector

We actually walk through a coil of a large number of turns, connected to a capacitor, tuned so that the circuit is in resonance. If one is carrying metallic objects, the inductance will change, and the resonance condition will no longer be applicable. This will cause a change in the current in the circuit, triggering an alarm.

Quality factor Q

The amplitude of the current in the series *LCR* circuit is given by

$$I_0 = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

and is maximum when

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

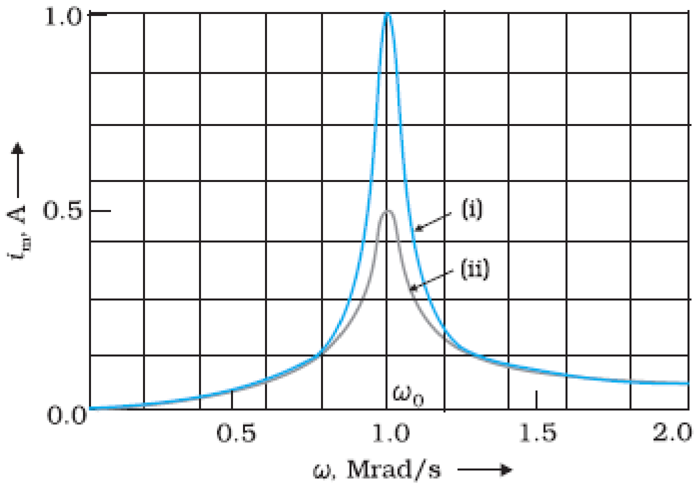
The current amplitude at this time will be

$$I_0 = \frac{V_0}{R}$$

For values of frequencies other than f_0, the amplitude of the current is less than the maximum value.

Suppose we choose a value of 'f' or angular frequency ω for which the current amplitude is $\frac{1}{\sqrt{2}}$ times its maximum value.

At this value, the power dissipated by the circuit becomes half. Study the graph given below:



From the graph we see that there are two vales of ω say ω_1 and ω_2, one greater than and the other smaller than ω_0 (at which we get maximum current).We may write

$$\omega_1 = \omega_0 + \Delta\omega$$

$$\omega_2 = \omega_0 - \Delta\omega$$

The difference $\omega_1 - \omega_2 = 2\Delta\omega$ is called **band width**

The smaller the value of bandwidth the sharper the resonance.

The sharpness of resonance is given by,

$$\frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R}$$

$\frac{\omega_0 L}{R}$ **is called the quality factor.**

It may also be defined as the ratio of voltage developed across inductance or capacitance at resonance to the voltage developed across the resistance .The voltage across R , at resonance, equals as we know to the applied voltage .

That is the reason we have placed $Q = \frac{(\omega_0 L)I}{RI} = \frac{\omega_0 L}{R}$

Also

$2\Delta v = \frac{\omega_0}{Q}$

So, larger the value of Q, the smaller is the value of 2Δω or the bandwidth and sharper

is the resonance.

Using $\omega_0^2 = \frac{1}{LC}$, can be equivalently expressed as $Q = 1/\omega_0 CR$.

We can express Q factor in different ways

Q= $\frac{\omega_0 L}{R}$

Q=(1/ω_0)CR.

Q = $\frac{\omega_0}{2\Delta\omega}$

Q= $\frac{1}{R}\sqrt{\frac{L}{C}}$

Significance of Q factor: We see that if the resonance is less sharp, not only is the maximum current less, the circuit is close to resonance for a larger range Δω of frequencies and the tuning of the circuit will not be good.

So, we see that

- **Less sharp the resonance less is the selectivity of the circuit or vice versa.**
- **If quality factor is large, i.e., *R* is low or *L* is large the circuit is more selective**
- **For resonance condition $X_C = X_L$ circuit becomes resistive and maximum current flows through it.**
- **The phase difference between current and voltage reduces to zero.**

Example:

Obtain the resonant frequency ω_0 of a series LCR circuit with L=2.0H, C=32μf and R = 10 ohms. What is the Q value of this circuit?

Solution:

L=2.0H, C=32 μf and R = 10 ohms

The resonant frequency $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.0 \times 32 \times 10^{-6}}} = 125\, rad/s$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{2}{32\times10^{-6}}} = \mathbf{25}$$

Example:

A radio can tune over the frequency range of a portion of MW broadcast band: (800 kHz to 1200 kHz). If its *LC* circuit has an effective inductance of 200 μH, what must be the range of its variable capacitor?

[*Hint:* For tuning, the natural frequency i.e., the frequency of free oscillations of the *LC* circuit should be equal to the frequency of the radio wave.

Solution:

F_1=800 kHz = 800 × 10^3 Hz

F_2= 1200 kHz = 1200 × 10^3 Hz

L= 200 μ H = 200 × 10^{-6} H

The resonant frequency is given by

$$f = \frac{1}{2\pi\sqrt{LC}} \quad or \quad f^2 = \frac{1}{4\pi^2 LC}$$

$$C = \frac{1}{4\pi^2 L f^2}$$

$$C_1 = \frac{49}{4(22)^2 200\times10^{-6}\times\left(800\times10^3\right)^2} = 197.73\, pF$$

And we can similarly calculate C_2=87.88 pF

So the range of capacitor is 87.88 pF – 197.73 pF

Average Power in AC Circuits

A whole house energy monitor measures the energy used by appliances connected to the house mains. To understand how it does this, it is useful to know something about how appliances interact with the electrical system.

In previous part of the module we had the following conclusions as the instantaneous power dissipated in the resistor is

$p = I^2R = I_0^2 R sin^2 \omega t$

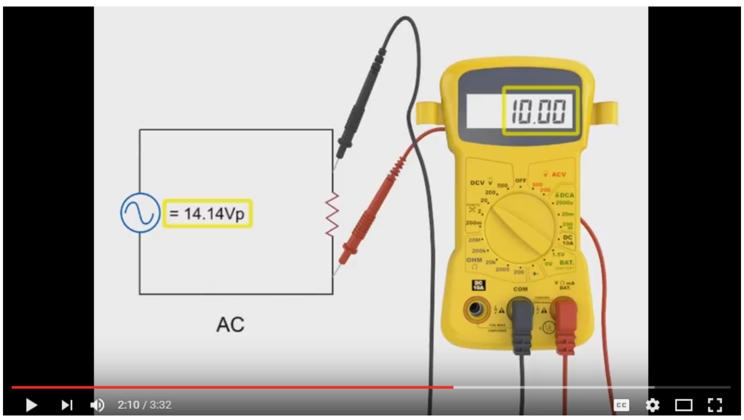
The average value of p over a cycle is

$= I_0^2 R$

The rms current is the equivalent dc current that would produce the same average power loss as the alternating current.

Source: Image Wikipedia

Power in an electric circuit is the rate of flow of energy past a given point of the circuit. In alternating current circuits, energy storage elements such as inductors and capacitors may result in periodic reversals of the direction of energy flow. The portion of power that, averaged over a complete cycle of the AC waveform, results in net transfer of energy in one direction is known as **active power** (sometimes also called real power). The portion of power due to stored energy, which returns to the source in each cycle, is known as **reactive power**.



https://www.youtube.com/watch?v=fEcqpxWZgco

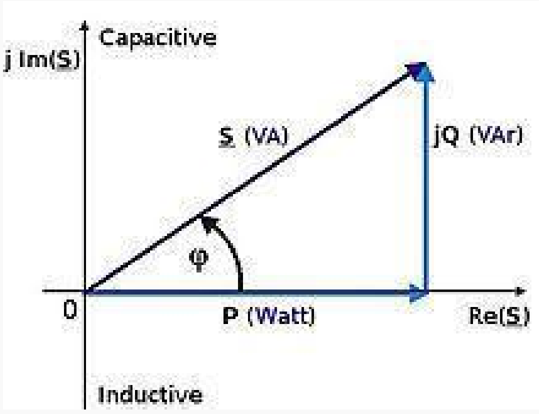
Active, Reactive and Apparent Power

In a simple alternating current (AC) circuit consisting of a source and a load , both the current and voltage are sinusoidal.

- If the **load is purely resistive**, the two quantities reverse their polarity at the same time. At every instant the product of voltage and current is positive or zero, with the result that the direction of energy flow does not reverse. **In this case, only active power is transferred.**
- If **the loads are purely reactive**, then the voltage and current are 90 degrees out of phase. For half of each cycle, the product of voltage and current is positive, but on the other half of the cycle, the product is negative, indicating that on average, exactly as much energy flows toward the load as flows back. There is no net energy flow over one cycle. In this case, only reactive power flows—there is no net transfer of energy to the load.
- Practical loads have resistance, inductance, and capacitance, so both active and reactive power will flow to real loads. Power engineers measure apparent power as the magnitude of the vector sum of active and reactive power.
- **Apparent power is the product of the root-mean-square of voltage and current.**

Electrical engineers take apparent power into account when designing and operating power systems, because though the current associated with reactive power does not work at the load, it heats the conductors and wastes energy. Conductors, transformers and generators must be sized to carry the total current, not just the current that does useful work. Failure to provide for the supply of sufficient reactive power in electrical grids can lead to lowered voltage levels and under certain operating conditions to the complete collapse of the network.

Conventionally, capacitors are considered to generate reactive power and inductors to consume it. If a capacitor and an inductor are placed in parallel, then the currents flowing through the inductor and the capacitor tend to cancel rather than add. This is the fundamental mechanism for controlling the power factor in electric power transmission; capacitors (or inductors) are inserted in a circuit to partially compensate for reactive power 'consumed' by the load. Purely capacitive circuits supply reactive power with the current waveform leading the voltage waveform by 90 degrees, while purely inductive circuits absorb reactive power with the current waveform lagging the voltage waveform by 90 degrees. The result of this is that capacitive and inductive circuit elements tend to cancel each other out.



Source: Image Wikipedia

In the diagram, P is the active power, Q is the reactive power (in this case positive), S is the complex power and the length of S is the apparent power. Reactive power does not do any work, so it is represented as the imaginary axis of the vector diagram. Active power does do work, so it is the real axis.

The unit for all forms of power is the watt (symbol: W), and conventionally expressed in volt-amperes (VA) since it is the product of rms voltage and rms current. The unit for reactive power is expressed as volt-ampere reactive. Since reactive power transfers no net energy to the load, it is sometimes called "wattless" power.

Expression for Power and Power Factor

We have seen that a voltage $V = V_0 sin\, \omega t$ applied to a series LCR circuit drives a current in the circuit given by $I = I_0 sin(\omega t + \phi)$

where

$I_0 = \frac{V_0}{Z}$ $and\ \phi = \left(\frac{X_C - X_L}{R}\right)$

Therefore, the instantaneous power, p supplied by the source is:

$$P = VI = \left(V = V_0 sin\, \omega t\right) \times \left(I = I_0 sin(\omega t + \phi)\right)$$

$$P = V_0 I_0\, sin\omega\, sin(\omega t + \phi)$$

$$P = \frac{V_0 I_0}{2} 2\, sin\, \omega\, sin(\omega t + \phi)$$

$$P = \frac{V_0 I_0}{2}[cos(\omega t - (\omega t + \phi)) - cos(\omega t + (\omega t + \phi))]$$

This is because 2 sinA sinB =cos(A−B)−cos(A+B)

$$P = \frac{V_0 I_0}{2}[\cos\phi - \cos(2\omega t + \phi)]$$

The average power over a cycle is given by the average of the two terms in R.H.S. of this equation.

It is only the second term which is time-dependent. Its average is zero (the positive half of the cosine cancels the negative half). Therefore,

$$P = \frac{V_0 I_0}{2}\cos\phi = \frac{V_0}{\sqrt{2}}\frac{I_0}{\sqrt{2}}\cos\phi$$

This can also be written as,

$$P = V_{rms} I_{rms} \cos\phi = VI\cos\phi$$

Or

P= $I^2 Z\cos\phi$

So, the average power dissipated depends not only on the voltage and current but also on the cosine of the phase angle ϕ between them.**The quantity cosϕ is called the Power Factor**.

The power factor value measures how much the mains efficiency is affected by both phase lag ϕ, and the harmonic content of the input current.

Let us consider the following cases

Case (i) **Resistive circuit**:

If the circuit contains only pure R, it is called a resistive circuit. In that case phase angle ϕ = 0, cos ϕ = 1. There is maximum power dissipation.

Case (ii) **purely inductive or capacitive circuit**:

If the circuit contains only an inductor or capacitor, we know that the phase difference between voltage and current is $\pi/2$. Therefore, cos ϕ = 0, and no power is dissipated even though a current is flowing in the circuit. **This current is referred to as wattles current.**

Case (iii) **LCR series circuit**:

In an LCR series circuit, power dissipated is given by

P= $V_{rms} I_{rms}\cos\phi$ = VI $\cos\phi$

where

$$\phi = \tan^{-1}(X_C - X_L)/R$$

So, ϕ may be non-zero in a RL or RC or LCR circuit. Even in such cases, power is dissipated only in the resistor.

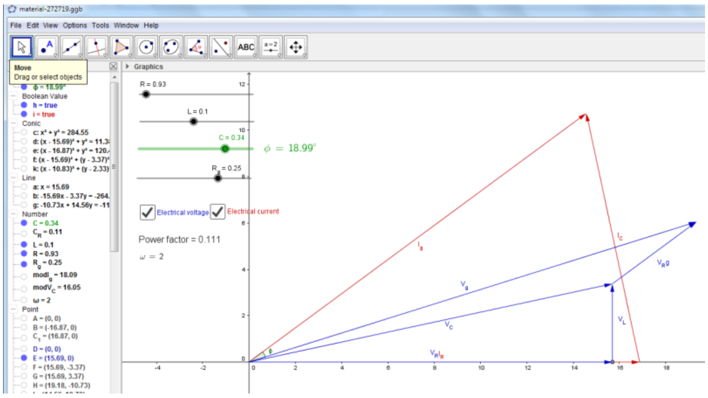
Case (iv) **Power dissipated at resonance in LCR circuit**:

At resonance ($X_C - X_L$) = 0, and ϕ = 0.

Therefore, cos ϕ = 1 and

$$P = I^2 Z = I^2 R.$$

That is, maximum power is dissipated in a circuit (through R) at resonance.



https://www.geogebra.org/m/EYYk8ksp?doneurl=%2Fsearch%2Fperform%2Fsearch%2FPOWER%2BFACTOR%2Fmaterials%2F

Significance of Power Factor:

The ratio of active power to apparent power in a circuit is called the power factor. For two systems transmitting the same amount of active power, the system with the lower power factor will have higher circulating currents due to energy that returns to the source from energy storage in the load. These higher currents produce higher losses and reduce overall transmission efficiency. A lower power factor circuit will have a higher apparent power and higher losses for the same amount of active power.

The power factor is one when the voltage and current are in phase. It is zero when the current leads or lags the voltage by 90 degrees. Power factors are usually stated as "leading" or "lagging" to show the sign of the phase angle of current with respect to voltage. Voltage is designated as the base to which current angle is compared, meaning that we think of current as either "leading" or "lagging" voltage.

Where the waveforms are purely sinusoidal, the power factor is the cosine of the phase angle (φ) between the current and voltage sinusoidal waveforms. Equipment data sheets and nameplates will often abbreviate power factor as " cosø" for this reason.

Example:

The active power is 700 W and the phase angle between voltage and current is 45.6°. Calculate:

a. The power factor
b. Apparent power

Solution:

The power factor is cos(45.6°) = 0.700.

The apparent power is then: 700 W / cos(45.6°) = 1000 VA.

Example:

(a) For circuits used for transporting electric power, a low power factor implies large power loss in transmission. Explain.

(b) Power factor can often be improved by the use of a capacitor of appropriate capacitance in the circuit. Explain.

Solution:

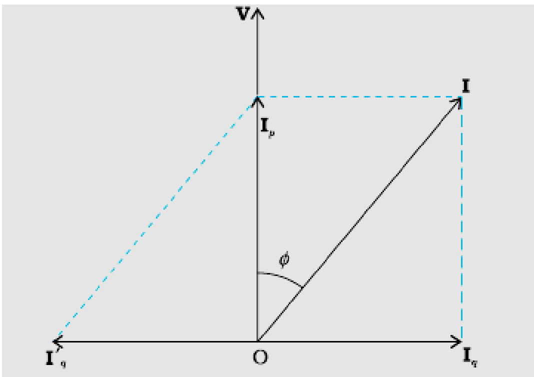
(a) We know that $P = I\ V \cos\phi$ where $\cos\phi$ is the power factor. To supply a given power at a given voltage, if $\cos\varphi$ is small, we have to increase current accordingly. But this will lead to large power loss $(I^2 R)$ in transmission.

(b) Suppose in a circuit, current I lags the voltage by an angle ϕ.

Then power factor $\cos\phi = R/Z$.

We can improve the power factor (tending to 1) by making Z tend to R.

Let us understand, with the help of a phasor diagram



Let us resolve I into two components. I_p along the applied voltage V and I_q perpendicular to the applied voltage. I_q as you have learnt, is called the wattless component since corresponding to this component of current, there is no power loss. I_P is known as the power component because it is in phase with the voltage and corresponds to power loss in the circuit.

It's clear from this analysis that if we want to improve power factor,

we must completely neutralize the lagging wattless current I_q by an equal leading wattless current I'_q. This can be done by connecting a capacitor of appropriate value in parallel so that I_q and I'_q cancel each other and P is effectively $I_p V$.

Example:

A sinusoidal voltage of peak value 283 V and frequency 50 Hz is applied to a series *LCR* circuit in which R = 3 W, *L* = 25.48 mH, and C = 796 μF.

Find

a. The impedance of the circuit;
b. The phase difference between the voltage across the source and the current;
c. The power dissipated in the circuit; and
d. The power factor.

Solution:

a. To find the impedance of the circuit, we first calculate X_L and X_C

$$X_L = 2 \times 3.14 \times 50 \times 25.48 \times 10^{-3} = 8 \text{ ohm}$$

$$X_C = \frac{1}{2 \times 3.14 \times 50 \times 796 \times 10^{=6}} = 4 \text{ ohm}$$

$$Z = \sqrt{3^2 + (8 - 4)^2}$$

$$= \mathbf{5\ ohm}$$

b. $\phi = \tan^{-1} (X_c - X_L)/ R$

$$= tan^{-1}\left(\frac{4-8}{3}\right) = -53.1^0$$

Since ϕ is negative, the current in the circuit lags the voltage across the source.

c. The power dissipated in the circuit is

$P = I^2R$

Now, $I = \frac{I_0}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\frac{283}{5}\right) = 40\ A$

Therefore $P = (40A)^2 \times 3 = 4800$ W

d. Power factor = $\cos \phi = \cos 53.1^0 = 0.6$

Think About These

a) **If a LC circuit is considered analogous to a harmonically oscillating spring block system, which energy of the LC circuit would be analogous to potential energy and which one analogous to kinetic energy?**
b) **Can the instantaneous power output of an ac source ever be negative? Can the average power output be negative?**
c) **What is a phasor?**
d) **Current in an AC circuit goes through a complete cycle in 0.1 second. Find the frequency of AC?**
e) **What is the basic unit of electrical power?**

Just for Information

<u>Advantages of AC over DC</u>

World over electric power stations generate and transmit AC power instead of DC

- **Some main advantages are listed below**
- **AC voltage generation is cheaper than DC.**
- **AC can be transmitted over long distances without loss.**
- **The peak voltage of AC can be increased or decreased using simple circuits (transformers).**
- **The magnitude of ac can be controlled using choke coils with low energy loss.**

<u>Disadvantages of AC over DC</u>

- **Since the peak value of AC is larger than its rms value = equivalent DC , AC is more dangerous as compared to DC.**
- **AC cannot be used for electroplating or in circuits where only D C is required.**
- **AC tends to cause drift in electrons on the surface of the conductor (skin effect) hence several thin insulated wires are used instead of a thick connecting wire used in DC circuits.**

Summary

We have learnt about LCR series circuit

- **Impedance**: Electrical impedance is the measure of the opposition that a circuit presents to a current when a voltage is applied. In quantitative terms, it is the complex ratio of the voltage to the current in an alternating current (AC) circuit. Z .this value of Z is the phasor sum of reactance and the resistance offered by the LCR at a particular frequency of source AC placed in the circuit

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

- **Reactance:** In electrical and electronic systems, reactance is the opposition of a circuit element to a change in current or voltage, due to that element's inductance or capacitance.

$$(X_L - X_C)$$

- **Phasors:** In Physics and Engineering, phasor is a complex number representing a sinusoidal function whose amplitude (A), angular frequency (ω), and initial phase (θ) are time-invariant. Basically, Phasors are rotating vectors.
- **Resonance:** A LCR circuit will be in resonance if $(X_L = X_C)$ depends upon L, C and f.
- **Resonance frequency ω_0:** The frequency of source for which the LCR circuit has a resistance = R, maximum current flows in the circuit.
- **Q factor:** It defines the sharpness of the maximum current.
- Band width For values of ω other than ω_0, the amplitude of the current is less than the maximum value. Suppose we choose a value of ω for which the current amplitude is 1/ 2 times its maximum value.($\omega_0 + \Delta\omega$) and ($\omega_0 - \Delta\omega$) At this value, the power dissipated by the circuit becomes half ,the difference between the two values of ω which will give half the maximum current is called band width.

$$Q = \frac{\omega_0}{2\Delta\omega}$$

- **Average Power**: It is the average amount of work done or energy converted per unit of time. The average power is often simply called "power".
- **Power Factor: (cosϕ)** In electrical engineering, the power factor of an AC electrical power system is defined as the ratio of the real power flowing to the load to the apparent power in the circuit, and is a dimensionless number in the closed interval of -1 to 1.
- **Choke Coil:** In electronics, a choke is an inductor used to block higher-frequency alternating current (AC) in an electrical circuit, while passing lower-frequency or direct current (DC).

- **Wattless Current:** Wattless current is AC component, whereby the power consumed in the circuit is zero.