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## Internal Resistance of a Cell

### Objectives

After going through this lesson, the learners will be able to:

- Know that potential difference is developed in chemical cells
- Interpreting the function of cells in circuits.
- Define Terminal potential difference of cell, internal resistance of a cell
- Derive a relation between EMF, TPD & internal resistance of a cell.
- Interpreting the need of combination of cells in series & parallel

### Content Outline

- Unit Syllabus
- Module breakup of unit syllabus
- Words you should know
- Introduction
- Need of cell in electrical circuit
- Electromotive Force(EMF)
- Terminal potential difference of a cell
- Internal resistance of a cell
- Factors affecting internal resistance of a cell
- Relation between EMF( $\epsilon$ ), TPD(V) and internal resistance (r)
- Combination of cells
- Need of combinations of cells
- Max. Power Transfer Theorem
- Solved Examples
- Questions for practice
- Summary

### Unit Syllabus

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Electric current, flow of electric charges in a metallic conductor, drift velocity and mobility, and their relation with electric current; Ohm's law' electrical resistance, V-I characteristics (linear & non- linear), electrical energy and power, electrical resistivity and conductivity.

Carbon resistors, colour code for carbon resistors; series and parallel combinations of resistors; temperature dependence of resistance

Internal resistance of a cell, potential difference and emf of cell, combination of cells in series and in parallel.

Kirchhoff's laws and simple applications; Wheatstone bridge, Meter bridge.

Potentiometer- principle and its applications to measure potential difference & for comparing emf of two cells; measurement of internal resistance of a cell.

### **Module Wise Distribution of Unit Syllabus - 08 Modules**

The above unit has been divided into 8 modules for better understanding.

Module 1	<ul style="list-style-type: none"><li>● Electric current,</li><li>● Solids liquids and gases</li><li>● Need for charge carriers speed of charge carriers in a metallic conductor</li><li>● Flow of electric charges in a metallic conductor</li><li>● Drift velocity,</li><li>● Mobility and their relation with electric current</li><li>● Ohm's law</li></ul>
Module 2	<ul style="list-style-type: none"><li>● Electrical resistance,</li><li>● V-I characteristics (linear and non-linear),</li><li>● Electrical energy and power,</li><li>● Electrical resistivity and conductivity</li><li>● Temperature dependence of resistance</li></ul>
Module 3	<ul style="list-style-type: none"><li>● Carbon resistors,</li><li>● Colour code for carbon resistors;</li><li>● Metallic Wire resistances</li><li>● Series and parallel combinations of resistors</li><li>● Grouping of resistances</li><li>● Current and potential differences in series and parallel circuits</li></ul>
Module 4	<ul style="list-style-type: none"><li>● Internal resistance of a cell,</li></ul>

	<ul style="list-style-type: none"> <li>● Potential difference and emf of a cell,</li> <li>● Combination of cells in series and in parallel.</li> <li>● Need for combination of cells</li> </ul>
Module 5	<ul style="list-style-type: none"> <li>● Kirchoff's Rules</li> <li>● Simple applications. of Kirchoff's Rules for calculating currents and voltages</li> <li>● Numerical</li> </ul>
Module 6	<ul style="list-style-type: none"> <li>● Wheatstone bridge</li> <li>● Balanced Wheatstone bridge condition derivation using Kirchoff's Rules</li> <li>● Wheatstone bridge and Metre Bridge.</li> <li>● Application of meter bridge</li> </ul>
Module 7	<ul style="list-style-type: none"> <li>● Potentiometer –</li> <li>● Principle</li> <li>● Applications to</li> <li>● Measure potential difference</li> <li>● Comparing emf of two cells;</li> <li>● Measurement of internal resistance of a cell.</li> <li>● Numerical</li> </ul>
Module 8	<ul style="list-style-type: none"> <li>● Numerical</li> <li>● Electrical energy and power</li> </ul>

## Module 4

### Words You Should Know

- **Potential drop across resistance:** It is the potential difference between ends of a resistance. In Ohm's law

$$V = IR$$

V is potential drop across resistance (for details see role of resistance in module 1)

- **Series combination:** When the same current flows through all the devices, it is said to be a series combination.
- **Parallel combination:** When all devices have the same potential difference, it is said to be a parallel combination.

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- **Electrical circuit:** It is the arrangement of electrical devices like resistance, cell, etc. to achieve a purpose /objective.
  - **Circuit components:** All devices that make up the circuit and are used for measurements of currents and voltages
  - **Chemical cell:** A cell using an electrolyte and electrodes to provide a potential difference between the electrodes due to chemical action

## **Introduction**

You must have observed that a cell-driven clock becomes slow after its long use or a torch lit by cells becomes dimmer when cells become old. Why is it so?

It is because the internal resistance of the cell increases after its long use.

Also you must have wondered how charge flows (current) in an electrical circuit when a cell is connected. This is because of the electromotive force (emf) of a cell.

Let us now recall what electromotive force (emf) and some other terms related to cell like Terminal potential difference (TPD), internal resistance are:

### **Electromotive Force (Emf) (E)**

It is the force which derives the charge in a closed path.

Sources of EMF are: - Cell, Generators, Dynamos.

## **Need for Cell in Electrical Circuit**

### **Why do we need a cell in a circuit?**

Charge can move on its own from higher to lower potential but cannot go on its own from lower to higher potential. So, to run charge in a closed path, we need a source EMF- a cell.

**Don't get confused that emf is forced . EMF is not a force but is work done in moving a unit positive charge once in a closed path.**

### **Emf of a Cell 'e'**

It is defined as the maximum potential difference between the terminals of the cell when no current is being drawn from the cell.

### **Terminal Potential Difference of a Cell (TPD) 'V'**

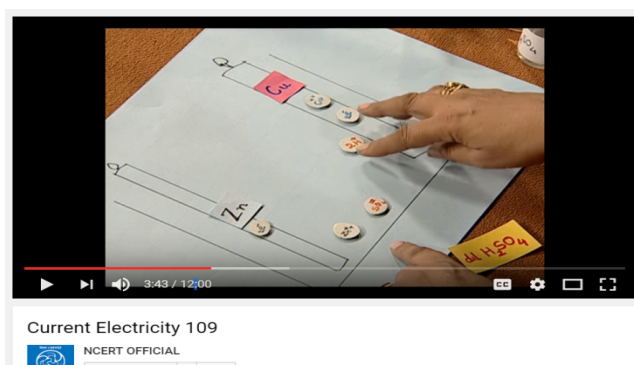
It is defined as the maximum potential difference between the terminals of a cell when current is being drawn from the cell.

The basic difference between emf and TPD of a cell is that in emf no current is being drawn from the cell but in TPD current is being drawn from the cell.

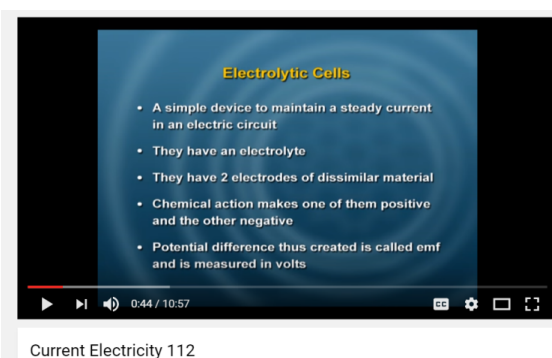
### Think About This

- Why does the potential difference between the terminals of a cell decrease when we use it in any circuit?
- Does the value of potential difference get restored when we remove it from the circuit
- Would the drop in potential difference between the electrodes be the same in all types of cells

### Internal Resistance of a Cell: (r)



<https://www.youtube.com/watch?v=fXnBfwWHASI&feature=youtu.be>



<https://www.youtube.com/watch?v=bribXH2eg20&feature=youtu.be>

A simple device to maintain a steady current in an electric circuit is the electrolytic cell. Basically a cell has two electrodes, called the positive (P) and the negative (N), the electrodes acquire positive or negative polarity due to chemical reaction. In other words, they are

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immersed in an electrolytic solution, dipped in the solution, and the electrodes exchange charges with the electrolyte.

The positive electrode has a potential difference  $V_+$  ( $V_+ > 0$ ) between itself and the electrolyte solution immediately adjacent to it marked Similarly the negative electrode develops a negative potential  $- (V_-)$  ( $V_- \geq 0$ ) relative to the electrolyte adjacent to it.

When there is no current, the electrolyte has the same potential throughout, so that the potential difference between P and N is  $V_+ - (-V_-) = V_+ + V_-$ . This difference is called the electromotive force (emf) of the cell and is denoted by  $\epsilon$ .

Thus  $\epsilon = V$

Note that  $\epsilon$  is actually a potential difference and not a force. The name emf, however, is used because of historical reasons, and was given at a time when the phenomenon was not understood properly.

To understand the significance of  $\epsilon$ , consider a resistor R connected across the cell. A current I flows across R .As explained before; a steady current is maintained because current flows from N to P through the electrolyte. Clearly, across the electrolyte the same current flows through the electrolyte but from N to P, whereas through R, it flows from P to N.

In practical calculations, internal resistances of cells in the circuit may be neglected when the current I is such that  $\epsilon \gg I r$ . The actual values of the internal resistances of cells vary from cell to cell. The internal resistance of dry cells, however, is much higher than the common electrolytic cells.

What kind of batteries do mobile phones have? Do you think they would have internal resistance? What about car batteries?

### **Factors Affecting Internal Resistance of a Cell**

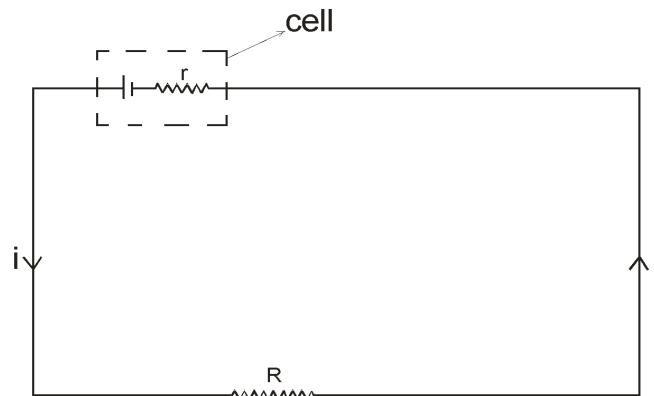
The internal resistance is the resistance offered by the electrodes and electrolytes of the cell to the current flowing.

**The value of internal resistance depends upon: -**

- Nature of electrodes
- Nature of electrolyte
- Concentration of electrolyte
- Distance between electrodes ( $d \propto r$ )

- Area of electrodes emerged in the electrolyte  
Increase area emerged, decrease in internal resistance for this reason, we pour water in cars or inverter batteries.
- Temperature of electrolyte: - Inversely proportional

### Relation Between Emf (E) and T.P.D. (V)



$$V = IR$$

By considering potential drop in the circuit and remembering conservation of energy

$$- IR - Ir + E = 0$$

$$- V - Ir + E = 0$$

$$V = E - Ir$$

For discharging of cell

$$E > V$$

**The above equation is for discharging of cells. Note that in discharging of cell emf > TPD**

Equation for charging of cell is given by

$$V = E + Ir$$

For charging of cell

$$V > E$$

**For charging of cell TPD > emf**

### Expression for internal resistance(r) of cell

$$\text{From } V = E - Ir$$

$$Ir = E - V$$

$$r = \frac{E - V}{I}$$

$r = R$

The maximum current that can be drawn from a cell is for  $R = 0$  and it is  $I_{\max} = \varepsilon/r$ . However, in most cells the maximum allowed current is much lower than this to prevent permanent damage to the cell.

### Combinations Of Cells

CELLS can be connected in **SERIES AND IN PARALLEL** like resistors; cells can be combined together in an electric circuit. And like resistors, one can, for calculating currents and voltages in a circuit, replace a combination of cells by an equivalent cell.

### Cells In Series

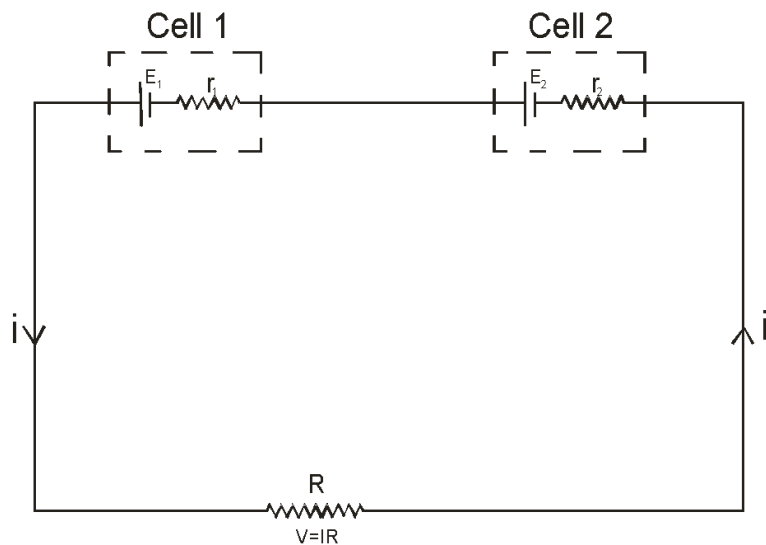


Fig: a showing  $E_1$  and  $E_2$  having

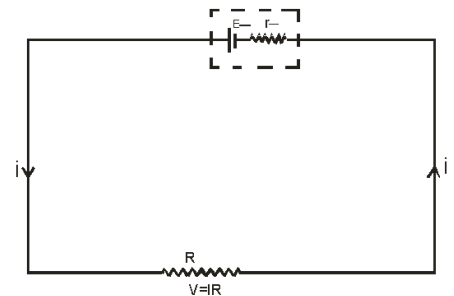


Fig: b showing  $E_{eq}$  &  $r_{eq}$

Internal resistances  $r_1$  and  $r_2$

$$- IR - Ir_2 + E_1 - Ir_2 + E_2 = 0$$

$$- V + (E_1 + E_2) - I(r_1 + r_2) = 0$$

$$V = (E_1 + E_2) - I(r_1 + r_2)$$

Comparing it with  $V = E_{eq} - I r_{eq}$



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$$E_{eq} = E_1 + E_2$$

$$r_{eq} = r_1 + r_2$$

Why should we compare it with  $V = E_{eq} - I r_{eq}$  ?

It is because the two cells  $E_1$  and  $E_2$  combine to form an equivalent cell ( $E_{eq}$ ), discharging equation of which is given by

$$V = E_{eq} - I r_{eq}$$

If there are 'n' identical cells in series

$$E_{eq} = nE$$

$$r_{eq} = nr$$

$$V = nE - I nr \quad IR = nE - nI r$$

$$I = \frac{nE}{R + nr}$$

I will be max if  $R \gg nr$

$$I_{\max} = \frac{nE}{R}$$

Max current can be drawn from a series combination, if external resistance is much greater than total internal resistance of cells.

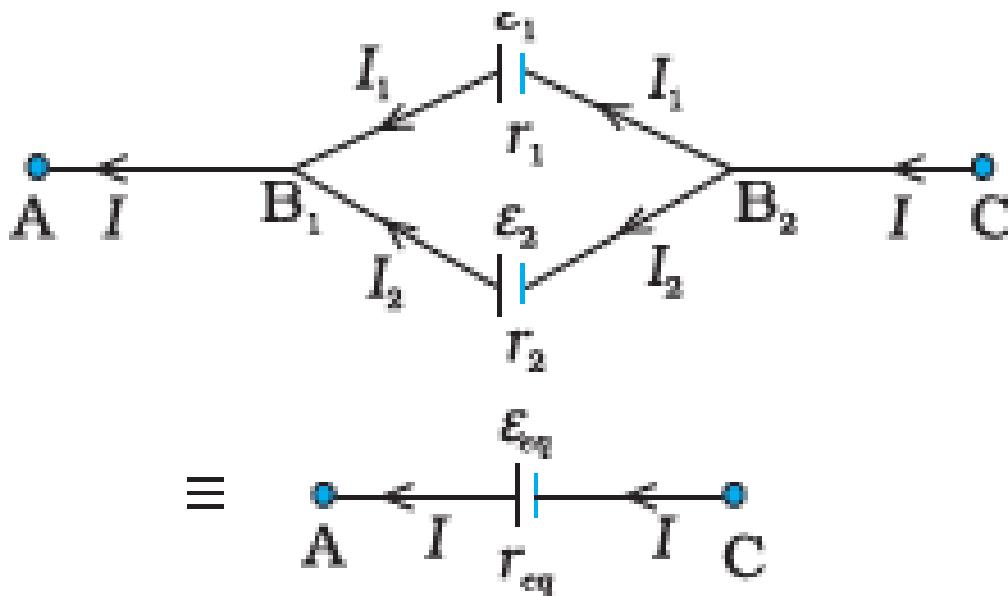
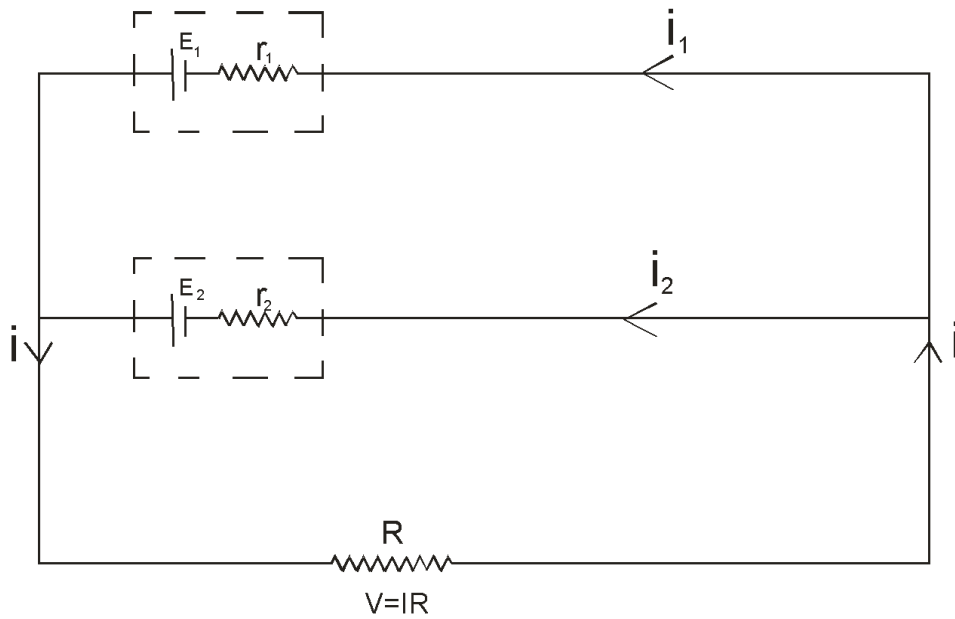
The rule for series combination clearly can be extended to any number of cells:

- i. The equivalent emf of a series combination of n cells is just the sum of their individual emf's, and
- ii. The equivalent internal resistance of a series combination of n cells is just the sum of their internal resistances.

This is so, when the current leaves each cell from the positive electrode.

If in the combination, the current leaves any cell from the *negative* electrode, the emf of the cell enters the expression for  $E_{eq}$  with a *negative* sign,

### Cells In Parallel



**Two cells in parallel. For connections across A and C, the combination can be replaced by one cell of emf  $\epsilon_{eq}$  and internal resistances  $r_{eq}$  whose values are given in**

$I_1$  and  $I_2$  are the currents leaving the positive electrodes of the cells. At the point  $B_1$ ,  $I_1$  and  $I_2$  flow in whereas the current  $I$  flow out. **Since as much charge flows in as out,**

We have

$$I = I_1 + I_2$$

Let  $V_{B_1}$  and  $V_{B_2}$  be the potentials at  $B_1$  and  $B_2$ , respectively.

Then, considering the first cell, the potential difference across its terminals is  $V_{(B_1)} - V_{(B_2)}$ .

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$$V = V(B_1) - V(B_2) = \epsilon_1 - I_1 r_1$$

Points  $B_1$  and  $B_2$  are connected exactly similarly to the second cell. Hence considering the second cell, we also have

$$V = V(B_1) - V(B_2) = \epsilon_2 - I_2 r_2$$

### Combining the last three equations

$$I = I_1 + I_2$$

Using  $V = \epsilon - IR$

$$I = \frac{\epsilon_1 - V}{r_1} + \frac{\epsilon_2 - V}{r_2} = \left( \frac{\epsilon_1}{r_1} + \frac{\epsilon_2}{r_2} \right) - V \left( \frac{1}{r_1} + \frac{1}{r_2} \right)$$

Hence  $V$  is given by

$$V = \frac{\epsilon_1 r_1 + \epsilon_2 r_2}{r_1 + r_2} - I \frac{r_1 r_2}{r_1 + r_2}$$

If we want to replace the combination by a single cell, between  $B_1$  and  $B_2$ , of emf  $\epsilon_{eq}$  and internal resistance  $r_{eq}$ ,

we would have

$$V = \epsilon_{eq} - I r_{eq}$$

The last two equations should be the same and hence

$$\epsilon_{eq} = \frac{\epsilon_1 r_1 + \epsilon_2 r_2}{r_1 + r_2}$$

and

$$I r_{eq} = I \frac{r_1 r_2}{r_1 + r_2}$$

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**We can put these in a simple way**

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

$$\frac{\epsilon_{eq}}{r_{eq}} = \frac{\epsilon_1}{r_1} + \frac{\epsilon_2}{r_2}$$

we had joined the positive terminals together and similarly the two negative ones, so that the currents  $I_1, I_2$  flow out of positive terminals.

If the negative terminal of the second is connected to positive terminal of the first, **The equations would still be valid with**

$$\epsilon_2 \rightarrow -\epsilon_2$$

**If there are n cells of emf  $\epsilon_1, \dots, \epsilon_n$  and of internal resistances  $r_1, \dots, r_n$  respectively,** connected in parallel, the combination is equivalent to a single cell of emf  $\epsilon_{eq}$  and internal resistance  $r_{eq}$ ,

such that

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \dots + \frac{1}{r_n}$$

$$\frac{\epsilon_{eq}}{r_{eq}} = \frac{\epsilon_1}{r_1} + \frac{\epsilon_2}{r_2} + \dots + \frac{\epsilon_n}{r_n}$$

### **Optional method**

**By Kirchhoff's junction law, , we will learn this in the next module**

**At junctions C and F**

$$\mathbf{I = I_1 + I_2}$$

**For loop ABCDEFA**

$$\mathbf{-E_1 + I_1 r_1 + IR = 0}$$

$$\mathbf{-E_1 + I_1 r_1 + V = 0}$$

$$\mathbf{I_1 = \frac{E_1 - V}{r_1}}$$

**In loop FEDCF**

$$\mathbf{-IR - I_2 r_2 + E_2 = 0}$$

$$\mathbf{-V - I_2 r_2 + E_2 =}$$

$$I_2 = \frac{E_2 - V}{r_2}$$

Putting  $I_1$  and  $I_2$  in (1)

$$I = \frac{E_1 - V}{r_1} + \frac{E_2 - V}{r_2}$$

$$I = \frac{E_1 r_2 - V r_2 + E_2 r_1 - V r_1}{r_1 r_2}$$

$$I r_1 r_2 = (E_1 r_2 + E_2 r_1) - V(r_1 + r_2)$$

Dividing equation by  $(r_1 + r_2)$  to bring it in standard form of discharging equation

$$V = E - I r$$

$$I \left( \frac{r_1 r_2}{r_1 + r_2} \right) = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} - V$$

Comparing it with  $V = E_{eq} - I r_{eq}$

$$E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$$

$$\Rightarrow \frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

If there are  $n$  identical cells in parallel

$$E_{eq} = E \quad (E_1 = E_2 = E = \dots, r_1 = r_2 = r)$$

$$\frac{1}{r_{eq}} = \frac{1}{r} + \frac{1}{r} + \dots n \text{ times}$$

$$r_{eq} = \frac{r}{n}$$

$$V = E_{eq} - I r_{eq}$$

$$I r = E - \frac{I r}{n}$$

$$I = \frac{E}{R + \frac{r}{n}}$$

I will be max if  $\frac{r}{n} \gg R$

$$I_{max} = \frac{E}{\frac{r}{n}} = \frac{nE}{r}$$

**Need Of Combinations Of Cells**

- Maximum current can be drawn from parallel combination if total internal resistance is much greater than total external resistance.
- Max current can be drawn from a series combination, if external resistance is much greater than total internal resistance of cells.

### Maximum Power Transfer Theorem:-

Maximum power can be transferred from combinations of cell to the external resistance if external resistance is equal to total internal resistance of the cell.

$$R = r_{eq}$$

### Solved Examples

#### Example

A cell of emf 2V and internal resistance  $0.1\Omega$  is connected to a  $3.9\Omega$  external resistance. What will be the potential difference across the terminals of the cell?

#### Solution

$$\text{Here } \varepsilon = 2\text{V}, \quad r = 0.1\Omega, \quad R = 3.9\Omega$$

$$V = \varepsilon - Ir$$

Or

$$IR = \varepsilon - Ir$$

$$I = \varepsilon / R+r = 2 / 3.9+0.1 = 0.5\text{A}$$

P.D. across the terminals of the cell,

$$V = IR = 0.5 \times 3.9 = 1.95\text{V}$$

#### Example

A dry cell of emf 1.6V and internal resistance  $0.10\Omega$  is connected to a resistance of R ohm. The current drawn from the cell is 2.0A. Find the voltage drop across R.

#### Solution

$$\text{Here } \varepsilon = 1.6\text{V}, \quad r = 0.10\Omega, \quad I = 2.0\text{A}$$

Voltage drop across R will be

$$V = \varepsilon - Ir$$

$$= 1.6 - 2.0 \times 0.10 = 1.4\Omega$$

#### Example

A battery of emf ' $\epsilon$ ' and internal resistance ' $r$ ' gives a current of 0.5A with an external resistor of  $12\Omega$  and a current of 0.25A with an external resistor of  $25\Omega$ . Calculate (i) internal resistance of the cell and (ii) emf of the cell.

**Solution**

EMF of the cell,  $\epsilon = I (R+r)$

In first case,  $\epsilon = 0.5(12+r)$

In second case,  $\epsilon = 0.25(25+r)$

Therefore,  $0.5(12+r) = 0.25(25+r)$

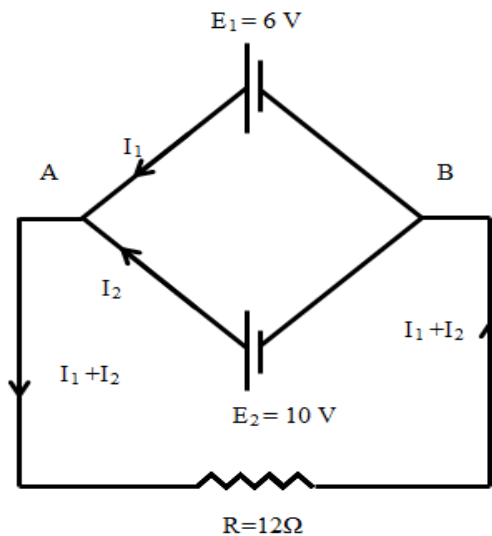
On solving, we get  $r = 1\Omega$

Hence,  $\epsilon = 0.5(12+1) = 6.5V$

**Example**

A battery of 6V & internal resistance  $0.5\Omega$  is joined in parallel with another of 10V & internal resistance  $1\Omega$ . The combination sends a current through an external resistance of  $12\Omega$ .

Find the current through each battery.



**Solution**

The arrangement of two batteries and external resistance is shown in the figure.

Let  $I_1$  &  $I_2$  be the currents given by the two batteries so that the current through the external resistance is  $(I_1 + I_2)$  as shown in the figure.

Applying Kirchhoff's 2<sup>nd</sup> rule to the closed circuit ARBE<sub>1</sub>A,

$$(I_1 + I_2) \times 12 + I_1 \times 0.5 - 6 = 0$$

Or  $12.5I_1 + 12I_2 = 6$  ..... (i)

Applying Kirchoff's 2<sup>nd</sup> rule to the closed circuit ARBE<sub>2</sub>A, we get

$$(I_1 + I_2) \times 12 + I_2 \times 1 - 10 = 0$$

Or  $12I_1 + 13I_2 = 10$  ..... (ii)

Multiplying (i) by 13 & (ii) by 12, we get

$$162.5I_1 + 156I_2 = 78$$
 ..... (iii)

$$144I_1 + 156I_2 = 120$$
 ..... (iv)

Subtracting (iv) from (iii), we get

$$18.5I_1 = -42 \quad \text{or} \quad I_1 = -42/18.5 = -2.27\text{A}$$

Negative sign shows that  $I_1$  actually flows in a direction opposite to what is shown in the figure

Substituting the value of  $I_1$  in (ii),

$$-12 \times 42/18.5 + 13I_2 = 10$$

Or  $13I_2 = 10 + 12 \times 42/18.5 = 185 + 504/18.5$

$$I_2 = 689/18.5 \times 13 = 2.86$$

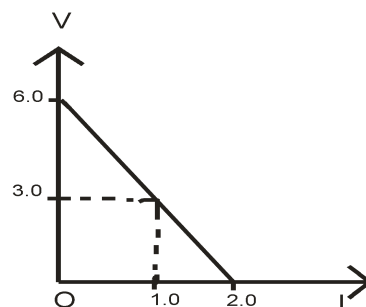
### Problems For Practice

- The reading on a high resistance voltmeter, when a cell is connected across it, is 2.2V. When the terminals of the cell are is connected to a resistance of 5Ω as shown in the figure, the voltmeter reading drops to 1.8V. Find the internal resistance of the cell.

(Answer:  $\frac{10}{9}\Omega$ )

### Example

The following graph shows the variation of terminal potential difference  $V$ , across a combination of three cells in series to a resistor, versus the current  $I$ .



(a) Calculate the emf of each cell



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(b) For what current  $I$ , will the power dissipation of the circuit be maximum.

**Answer:** (a) 2V (b) 1A

- Give expression for discharging of a cell. Which is greater, terminal p.d. or emf of a cell?
- A cell of emf  $E$  and internal resistance  $r$  gives the current of 2A in a resistance of  $12\Omega$  & gives a current of 1A in  $8\Omega$  external resistance. Find emf and internal resistance of a cell.

**Answer:**  $Emf = 8v$  and  $r = 16\Omega$

- Establish a relation between E.M.F. and potential difference of a cell.
- What is the difference between electromotive force and terminal voltage of a cell? How are they related to each other?
- What is the internal resistance of a cell due to?

(Internal resistance of a cell depends upon:

- (i) the nature, concentration and temperature of electrolyte,
- (ii) the nature of electrodes,
- (iii) the distance b/w the electrodes and
- (iv) area of electrodes immersed in electrolyte.)

**see video with this module**

### Summary

You have learnt in this module:

- **Electromotive Force (EMF):**  
It is the force which makes change to flow in an electrical circuit.  
It is defined as **Work done** in moving a unit positive charge once in a closed circuit.
- **EMF of a cell ( $\epsilon$ ):** It is the maximum potential difference between electrodes of a cell when no current is being drawn from the cell. emf is the potential difference between the electrodes of a cell is due to the chemical reaction taking place inside the cell emf is defined as work done in moving a unit +ve charge once in a closed circuit of cell is defined as maximum potential difference between electrodes of a cell when no current is being drawn from the cell
- **Terminal potential difference of a cell (V):** It is the max. Potential difference between electrodes of a cell when current is being drawn.

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● **Internal resistance of a cell(r):** It is the resistance offered by electrolyte to current flowing.

- Primary electrolytic cell and how potential difference is created in it by chemical action
- When the cell is in use or is placed in a circuit, the potential difference between the electrodes falls
- The new potential difference is called terminal potential difference
- Internal Resistance (r) of a cell is the obstruction offered by electrodes & electrolytes of a cell to current flowing through it.
- Internal resistance of a cell can be determined in the laboratory
- Internal resistance can be calculated

$$r = \left( \frac{E-V}{V} \right) R$$

- Cells can be combined in series and parallel
- The need for combination is to provide suitable voltage to a circuit with consideration to net circuit resistance internal
- Expression for two cells in series is given by:

$$V = (E_1 + E_2) - I(r_1 + r_2)$$

For n identical cells

$$V = nE - Inr$$

Expression for two cells in parallel is given by:

$$V = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} - I \left( \frac{r_1 r_2}{r_1 + r_2} \right)$$

For n identical cells

$$V = E - I \frac{r}{n}$$