MATHEMATICS - CLASS XII

Time : 3 Hours Max. Marks : 100

The weightage of marks over different dimensions of the question paper shall be as follows:

(C) Scheme of Option:

 There is no overall choice. However, an internal choice in four questions of four marks each and two questions of six marks each has been provided.

Section—A

Choose the correct answer from the given four options in each of the Questions 1 to 3.

- **1.** If $\begin{bmatrix} x y \\ x z \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} -2 \\ 2 \end{bmatrix}$, then (x, y) is **DESIGN OF THE QUE**
 2 1 $\left[\begin{array}{c} 1 \\ -2 \end{array}\right]$, then (x, y) is

(B) $(1, -1)$

(D) $(-1, -1)$ **Section—A**
 Section—A

t answer from the given four options in each of the
 $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then (x, y) is

(B) $(1, -1)$

(D) $(-1, -1)$

the triangle with vertices $(-2, 4)$, $(2, k)$ and $(5, 4$ **Section—**

he correct answer from the given four of
 $\begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then (x, y) is

(1, 1) (B) (1, -1)

(-1, 1) (D) (-1, -1) **Section—

he correct answer from the given four of
** $\begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ **, then** (x, y) **is

(1, 1) (B) (1, -1)

(-1, 1) (D) (-1, -1)

area of the triangle with vertices (-2, 4) Section—A**

the correct answer from the given four options in each
 $\begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then (x, y) is

(B) $(1, -1)$

(D) $(-1, -1)$ **Section—A**

the correct answer from the given four options in each of the C
 $\begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then (x, y) is

(1,1) (B) (1,-1)

(-1, 1) (D) (-1, -1)

e area of the triangle wi If $\begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then (x, y) is

(A) (1, 1) (B) (1, -1)

(C) (-1, 1) (D) (-1, -1) If $\begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then (x, y) is

(A) (1, 1) (B) (1, -1)

(C) (-1, 1) (D) (-1, -1)

The area of the triangle with vertices (-2, 4) - y $\left[4 \quad 3 \right] [-2]$, then (x, y) is

(1, 1) (B) (1, -1)

(-1, 1) (D) (-1, -1)

rea of the triangle with vertices (-2

of k is

4 (B) -2

6 (D) -6

ne y = x + 1 is a tangent to the curv

(1, 2) (B) (2, 1)

(1, -2) (D) (- $\begin{vmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then (x, y)
 i j $\begin{vmatrix} 1 \\ -2 \end{vmatrix}$, then (x, y)
 i $\begin{vmatrix} 0 \\ -1 \end{vmatrix}$, if $\begin{vmatrix} 0 \\ -2 \end{vmatrix}$
 i $\begin{vmatrix} 0 \\ -6 \end{vmatrix}$
 i $\begin{vmatrix} 0 \\ -2 \end{vmatrix}$
 i $\begin{vmatrix} 0 \\ -2 \end$ If $\begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then (x, y) is

(A) (1, 1) (B) (1, -1)

(C) (-1, 1) (D) (-1, -1)

The area of the triangle with vertices (-2, 4)

value of k is

(A) 4 (B) -2

(C) 6 (D) -6
 i j i j $(x+y) = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then (x, y) is

(1, 1) (B) (1, -1)

(-1, 1) (D) (-1, -1)

(-1, 1) (D) (-1, -1)

area of the triangle with vertices (-2, 4), (2, k) and (5, 4) is

e of k is

4 (B) -2

(D) -
- **2.** The area of the triangle with vertices $(-2, 4)$, $(2, k)$ and $(5, 4)$ is 35 sq. units. The value of *k* is

- **3.** The line $y = x + 1$ is a tangent to the curve $y^2 = 4x$ at the point
- (A) 4 (B) -2

(C) 6 (D) -6

The line $y = x + 1$ is a tangent to the curve

(A) (1, 2) (B) (2, 1)

(C) (1, -2) (D) (-1, 2) (C) 6 (D) -6

The line $y = x + 1$ is a tangent to the curve

(A) (1, 2) (B) (2, 1)

(C) (1, -2) (D) (-1, 2)

Construct a 2 × 2 matrix whos
- **4.** Construct a 2 \times 2 matrix whose elements a_{ij} are given by

$$
a_{ij} = \begin{cases} \frac{\left|-3\hat{i} + j\right|}{2}, & \text{if } i \neq j\\ (i + j)^2, & \text{if } i = j. \end{cases}
$$

- **5.** Find the value of derivative of tan⁻¹ (*e*^x) w.r.t. *x* at the point $x = 0$.
- 6. The Cartesian equations of a line are $\frac{x^2-2}{2} = \frac{y^2-2}{-5} = \frac{z^2-2}{3}$. Find the vector equation 2, 4), (2, k) and (5, 4) is 35 sq. units. The

ve $y^2 = 4x$ at the point

nose elements a_{ij} are given by
 $y = 0$.
 $y =$ curve $y^2 = 4x$ at the point
whose elements a_{ij} are given by
 y^2) w.r.t. x at the point $x = 0$.
 $\frac{-3}{2} = \frac{y+2}{-5} = \frac{z-6}{3}$. Find the vector equation *z* $(-2, 4)$, $(2, k)$ and $(5, 4)$ is 35 sq. units. The
 x $y^2 = 4x$ at the point
 x a_{ij} are given by
 x (e^x) w.r.t. *x* at the point $x = 0$.
 $x - 3 = \frac{y + 2}{-5} = \frac{z - 6}{3}$. Find the vector equation 4), (2, k) and (3, 4) is 33 sq. units. The

ve $y^2 = 4x$ at the point

sse elements a_{ij} are given by

w.r.t. x at the point $x = 0$.
 $=\frac{y+2}{-5} = \frac{z-6}{3}$. Find the vector equation of the line. 3. The line $y = x + 1$ is a tangent to the curve $y^2 = 4x$

(A) (1, 2) (B) (2, 1)

(C) (1, -2) (D) (-1, 2)

4. Construct a 2 × 2 matrix whose elen
 $a_{ij} = \begin{cases} \frac{|-3\hat{i} + j|}{2}, & \text{if } i \neq j \\ (i + j)^2, & \text{if } i = j. \end{cases}$

5. Find the *x* = *x* + 1 is a tangent to the curve $y^2 = 4x$ at the point

2) (B) (2, 1)

-2) (D) (-1, 2)

t a 2 × 2 matrix whose elements a_{ij}
 $\left(\frac{x+j}{2}\right)$, if $i \neq j$
 j)², if $i = j$.

value of derivative of tan⁻¹ (e^v)
- $\int\limits_{-\pi}^{\pi}$

Fill in the blanks in Questions 8 to 10.

$$
8. \qquad \int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} dx =
$$

- MATHEMATICS
the blanks in Questions 8 to 10.
 $\frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} dx =$
 $\vec{a} = 2\hat{i} + 4\hat{j} \hat{k}$ and $\hat{b} = 3\hat{i} 2\hat{j} + \lambda \hat{k}$ are *x x* μ *x* $\$ 308 MATHEMATICS

Fill in the blanks in Questions 8 to 10.

8. $\int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} dx =$

9. If $\vec{a} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\hat{b} = 3\hat{i} - 2\hat{j} + \lambda \hat{k}$ are perpendicular to each other, the
 $=$

10. The projection of 9. If $\vec{a} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\hat{b} = 3\hat{i} - 2\hat{j} + \lambda\hat{k}$ are perpendicular to each other, then MATHEMATICS

the blanks in Questions 8 to 10.
 $\int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} dx =$

If $\vec{a} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\hat{b} = 3\hat{i} - 2\hat{j} + \lambda \hat{k}$ are perpen
 $=$

The projection of $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$ along $\hat{b} = 2\hat{i} - 3$ ³⁰⁸ MATHEMATICS
 10. THII in the blanks in Questions 8 to 10.
 10. If $\vec{a} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\hat{b} = 3\hat{i} - 2\hat{j} + \lambda \hat{k}$ are perpendicular to each other, then
 $\vec{a} = \frac{1}{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}$
 11. Prove 308 MATHEMATICS

Fill in the blanks in Questions 8 to 10.

8. $\int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} dx =$

9. If $\vec{a} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\hat{b} = 3\hat{i} - 2\hat{j} + \lambda \hat{k}$ are perpendicular to each other,
 $=$
 10. The projection of The same of $\hat{A} \hat{j} - \hat{k}$ and $\hat{b} = 3\hat{i} - 2\hat{j} + \lambda \hat{k}$ are perpendicular to each other, then

con of $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$ along $\hat{b} = 2\$ stions 8 to 10.

In $\hat{b} = 3\hat{i} - 2\hat{j} + \lambda \hat{k}$ are perpendicular to each other, then
 $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$ along $\hat{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ is

 Section --B

 I+sin x + $\sqrt{1 - \sin x}$ $\begin{cases} \frac{x}{2}, & 0 < x < \frac$ 8 to 10.
 $3\hat{i} - 2\hat{j} + \lambda \hat{k}$ are perpendicular to each other, t
 $3\hat{j} + \hat{k}$ along $\hat{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ is
 Section —B
 $\frac{\overline{x} + \sqrt{1 - \sin x}}{\overline{x} - \sqrt{1 - \sin x}} = \frac{x}{2}, \qquad 0 < x < \frac{\pi}{2}$

OR 8 to 10.

3 $\hat{i} - 2\hat{j} + \lambda \hat{k}$ are perpendicular to each oth

3 $\hat{j} + \hat{k}$ along $\hat{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ is
 Section—B
 $\frac{x + \sqrt{1 - \sin x}}{x - \sqrt{1 - \sin x}}\Big| = \frac{x}{2}, \quad 0 < x < \frac{\pi}{2}$

OR

f sin⁻¹x + sin⁻¹2x = $\frac{\pi}{3}$, uestions 8 to 10.

= _____
 $\hat{\epsilon}$ and $\hat{b} = 3\hat{i} - 2\hat{j} + \lambda \hat{k}$ are perpendicular to each other, then

of $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$ along $\hat{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ is

Section — B
 $\left[\frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x}$ vestions 8 to 10.

= _____
 $\hat{\epsilon}$ and $\hat{b} = 3\hat{i} - 2\hat{j} + \lambda \hat{k}$ are perpendicular to each other, then

of $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$ along $\hat{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ is ______

Section-B
 $\left[\frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1$ + − − *b* $= 3\hat{i} - 2\hat{j} + \lambda \hat{k}$ are perpendicular to each other, then
 $= \hat{i} + 3\hat{j} + \hat{k}$ along $\hat{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ is
 Section—**B**
 $\frac{\sin x + \sqrt{1 - \sin x}}{\sin x - \sqrt{1 - \sin x}} = \frac{x}{2}, \quad 0 \le x \le \frac{\pi}{2}$

OR
 $\text{or } x \text{ if } \sin^{-1}x + \sin$ $\hat{b} = 3\hat{i} - 2\hat{j} + \lambda \hat{k}$ are perpendicular to each other, then
 $\hat{i}+3\hat{j}+\hat{k}$ along $\hat{b}=2\hat{i}-3\hat{j}+6\hat{k}$ is
 Section—B
 $\frac{\sin x + \sqrt{1-\sin x}}{\sin x - \sqrt{1-\sin x}}\Big|_{\frac{x}{2}} \times 0 < x < \frac{\pi}{2}$

OR

x if $\sin^{-1}x + \sin^{-1}2x = \frac{\pi}{3}$,
- 10. The projection of $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$ along $\hat{b} = 2\hat{i} 3\hat{j} + 6\hat{k}$ is

Section-

11. Prove that
$$
\cot^{-1} \left\{ \frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}} \right\} = \frac{x}{2}
$$
, $0 < x < \frac{\pi}{2}$

$$
\mathbf{OR} \qquad \qquad
$$

Solve the equation for *x* if $\sin^{-1}x + \sin^{-1}2x =$ π > 0

12. Using properties of determinants, prove that

$$
\vec{a} = \hat{i} + 3\hat{j} + \hat{k} \text{ along } \hat{b} = 2\hat{i} - 3\hat{j} + 6\hat{k} \text{ is}
$$
\n**Section**—**B**
\n
$$
\frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}} = \frac{x}{2}, \quad 0 < x < \frac{\pi}{2}
$$
\nOR
\n1 for x if sin⁻¹x + sin⁻¹2x = $\frac{\pi}{3}$, x > 0
\nIf determinants, prove that
\n
$$
\begin{vmatrix}\nb + c & c + a & a + b \\
q + r & r + p & p + q \\
y + z & z + x & x + y\n\end{vmatrix} = 2 \begin{vmatrix} a & b & c \\
 p & q & r \\
x & y & z \end{vmatrix}
$$
\nunity of the function f given by $f(x) = |x + 1| + |x + 2|$ at $x = -1$ and

13. Discuss the continuity of the function *f* given by $f(x) = |x+1| + |x+2|$ at $x = -1$ and $x = -2$. $x < \frac{\pi}{2}$
 $x > 0$
 b c
 q r
 y z
 $(x) = |x+1| + |x+2|$ at $x = -1$ and
 $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$. OR

OR
 $\int \sin^{-1}2x = \frac{\pi}{3}$, $x > 0$

ove that
 $\int \tan^{-1}2x = \frac{\pi}{3}$, $x > 0$

ove that
 $\int \tan^{-1}2x = \frac{\pi}{3}$, $x > 0$
 $\int \tan^{-1}2x = \frac{\pi}{3}$
 $\int \tan^{-1}2x = \frac{\pi}{3}$
 $\int \tan^{-1}2x = -1$

14. If
$$
x = 2\cos\theta - \cos2\theta
$$
 and $y = 2\sin\theta - \sin2\theta$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$.

If
$$
x\sqrt{1+y} + y\sqrt{1+x} = 0
$$
, prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$, where $-1 < x < 1$

DESIGN OF THE QUESTION PAPER
 $\frac{-1}{(1+x)^2}$, where $-1 < x < 1$

eep. Water is poured into it at the

ne water level rising at the instar **EXECT OF THE QUESTION PAPER** 309
 $\sqrt{1 + y} + y\sqrt{1 + x} = 0$, prove that $\frac{dy}{dx} = \frac{-1}{(1 + x)^2}$, where $-1 < x < 1$

Le is 10cm in diameter and 10cm deep. Water is poured into it at the rate of ic cm per minute. At what rate is **EXECT OF THE QUESTION PAPER** 309
 $x\sqrt{1+y} + y\sqrt{1+x} = 0$, prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$, where $-1 < x < 1$

cone is 10cm in diameter and 10cm deep. Water is poured into it at the rate of

ubic cm per minute. At what rate DESIGN OF THE QUESTION PAPER 309
 $+y + y\sqrt{1 + x} = 0$, prove that $\frac{dy}{dx} = \frac{-1}{(1 + x)^2}$, where $-1 < x < 1$

is 10cm in diameter and 10cm deep. Water is poured into it at the rate of

cm per minute. At what rate is the water le **15.** A cone is 10cm in diameter and 10cm deep. Water is poured into it at the rate of 4 cubic cm per minute. At what rate is the water level rising at the instant when the depth is 6cm? DESIGN OF THE QU

DESIGN OF THE QU
 $\sqrt{1 + x} = 0$, prove that $\frac{dy}{dx} = \frac{-1}{(1 + x)^2}$, where $-1 <$

in diameter and 10cm deep. Water is poured

minute. At what rate is the water level rising

n?

OR

als in which the functi bESIGN
 $+ y\sqrt{1 + x} = 0$, prove that $\frac{dy}{dx} = \frac{-1}{(1 + x)^2}$,

0cm in diameter and 10cm deep. Wate

per minute. At what rate is the water 1

6cm?

OR

ervals in which the function f given t

ng (ii) decreasing
 $\frac{3x-2}{(x+3$ DESIGN OF THE QUES
 y√1+*x* = 0, prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$, where -1< *x*

cm in diameter and 10cm deep. Water is poured in

ver minute. At what rate is the water level rising a

6cm?

OR

rvals in which the fun bestors of the questron
 $-\frac{1}{x} + y\sqrt{1 + x} = 0$, prove that $\frac{dy}{dx} = \frac{-1}{(1 + x)^2}$, where $-1 < x < 1$

10cm in diameter and 10cm deep. Water is poured into

n per minute. At what rate is the water level rising at the

is 6cm cm in diameter and 10cm deep. Wat

ber minute. At what rate is the water

6cm?

OR

rvals in which the function f given

g (ii) decreasing
 $\frac{3x-2}{x+3)(x+1)^2}$ dx

OR
 $\log(\log x) + \frac{1}{\log x)^2} dx$
 $\frac{x \sin x}{x + \cos^2 x} dx$ ter and 10cm deep. Water is pool to the function deep. Water is positive to the function dependent of given by $f(x)$ sing $\left(\frac{dx}{\log x}\right)^2 dx$ ameter and 10cm deep. Water is poured into it at
the At what rate is the water level rising at the ins
OR
which the function f given by $f(x) = x^3 + \frac{1}{x^3}$, x
creasing
 $\frac{2}{x+1^2}$ dx
 $(x) + \frac{1}{\log x)^2} dx$

OR

Find the intervals in which the function *f* given by $f(x) = x^3 + \frac{1}{3}$, $+\overline{3}$, $x \neq 0$ is $1 \qquad Q$ x^3 , $x \sim 0.5$

(i) increasing (ii) decreasing

16. Evaluate
$$
\int \frac{3x-2}{(x+3)(x+1)^2} dx
$$

OR

Evaluate
$$
\int \left(\log (\log x) + \frac{1}{\log x} \right) dx
$$

- **17.** Evaluate $\int_{1+\cos^2 x}^{1+\sin x} dx$ $\int_{0}^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$ π
- OR

tervals in which the function f given

ong (ii) decreasing
 $\frac{3x-2}{(x+3)(x+1)^2} dx$

OR
 $\left[\log(\log x) + \frac{1}{\log x^2} \right] dx$
 $\frac{x \sin x}{1 + \cos^2 x} dx$

fferential equation of all the circles w

res lie on x-axis.

fferential equatio OR

rvals in which the function f given by $f(x) =$
 $\frac{3x-2}{(x+3)(x+1)^2}$ dx

OR
 $\log(\log x) + \frac{1}{\log x^2} dx$
 $\frac{x \sin x}{x \cos^2 x} dx$

rential equation of all the circles which pass

s lie on *x*-axis. **18**. Find the differential equation of all the circles which pass through the origin and whose centres lie on *x*-axis. Find the differential

vhose centres lie o

bolve the differential
 $2y dx - (x^3 + y^3) dx$ **20.** If $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}, \vec{a} \neq \vec{0}$ and $\vec{b} \neq \vec{c}$, show that $\vec{b} = \vec{a} \times \vec{c}$, $\vec{a} \neq \vec{0}$

20. If $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}, \vec{a} \neq \vec{0}$ and $\vec{b} \neq \vec{c}$, show that $\vec{b} = \vec{c} + \lambda \vec{a}$ OR

aluate $\int \left(\log(\log x) + \frac{1}{\log x)^2} \right) dx$

aluate $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$

aluate $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$

d the differential equation of all the circles which pass through the origin and

ose centres lie on *x*-axis OR

all the circles which pass through the origin and
 $\int dx$

show that $\vec{b} = \vec{c} + \lambda \vec{a}$ for some scalar λ . For some scalar λ .
 $\vec{b} = \vec{c} + \lambda \vec{a}$ for some scalar λ .
- **19.** Solve the differential equation

$$
x^2y\ dx - (x^3 + y^3)\ dy = 0
$$

$$
\vec{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (1 + \lambda)\hat{k}
$$
 and $\vec{r} = (1 - \vec{\mu})\hat{i} + (2\vec{\mu} - 1)\hat{j} + (\vec{\mu} + 2)\hat{k}$

21. Find the shortest distance between the lines
 $\vec{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (1 + \lambda)\hat{k}$ and $\vec{r} = (1 - \vec{\mu})\hat{i} + (2\vec{\mu} - 1)\hat{j} + (\vec{\mu} + 2)\hat{k}$

22. A card from a pack of 52 cards is lost. From the remaining cards of the car ATHEMATICS

the shortest distance between the lines
 $\lambda - 1\hat{i} + (\lambda + 1)\hat{j} - (1 + \lambda)\hat{k}$ and $\vec{r} = (1 - \hat{\mu})\hat{i} + (2\hat{\mu} - 1)\hat{j} + (\hat{\mu} + 2)\hat{k}$

and from a pack of 52 cards is lost. From the remaining cards of the pack, two

sa **22.** A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and found to be hearts. Find the probability of the missing card to be a heart. between the lines
 $(+\lambda)\hat{k}$ and $\vec{r} = (1 - \vec{\mu})\hat{i} + (2\vec{\mu} - 1)\hat{j} + (\vec{\mu} + 2)\hat{k}$

2 cards is lost. From the remaining cards of the pack, two

md to be hearts. Find the probability of the missing card

Section—C

and B be istance between the lines
 $\hat{j} - (1 + \lambda)\hat{k}$ and $\vec{r} = (1 - \vec{\mu})\hat{i} + (2\vec{\mu} - 1)\hat{j} + (\vec{\mu} + 2)\hat{k}$

of 52 cards is lost. From the remaining cards of the pack, two

d found to be hearts. Find the probability of the missing c se between the lines
 $(1+\lambda)\hat{k}$ and $\vec{r} = (1-\vec{\mu})\hat{i} + (2\vec{\mu}-1)\hat{j} + (\vec{\mu}+2)\hat{k}$

2 cards is lost. From the remaining cards of the pack, two

and to be hearts. Find the probability of the missing card
 Section—C

and B (1+ λ) \hat{k} and $\vec{r} = (1 - \vec{\mu})\hat{i} + (2\vec{\mu} - 1)\hat{j} + (\vec{\mu} + 2)\hat{k}$

52 cards is lost. From the remaining cards of the pack, two

und to be hearts. Find the probability of the missing card
 Section—C

and B be given by ance between the lines
 $-(1+\lambda)\hat{k}$ and $\vec{r} = (1-\vec{\mu})\hat{i} + (2\vec{\mu}-1)\hat{j} + (\vec{\mu}+2)\hat{k}$

f 52 cards is lost. From the remaining cards of the pack, two

found to be hearts. Find the probability of the missing card
 Section—C (1+ λ) \hat{k} and $\vec{r} = (1 - \vec{\mu})\hat{i} + (2\vec{\mu} - 1)\hat{j} + (\vec{\mu} + 2)\hat{k}$

52 cards is lost. From the remaining cards of the pack, two

und to be hearts. Find the probability of the missing card
 Section—C

and B be given b $\vec{r} = (2-1)\hat{i} + (2\hat{i}+1)\hat{j} - (1+\lambda)\hat{k}$ and $\vec{r} = (1-\hat{i}t)\hat{i} + (2\hat{i}t-1)\hat{j} + (\hat{i}t+2)\hat{k}$

A card from a pack of 52 cards is lost. From the remaining cards of the pack, two

cards are drawn and found to be hearts. Find the

Section—C

23. Let the two matrices A and B be given by

Verify that $AB = BA = 6I$, where I is the unit matrix of order 3 and hence solve the system of equations

24. On the set \mathbb{R} – $\{-1\}$, a binary operation is defined by

 $a * b = a + b + ab$ for all $a, b \in \mathbb{R} - \{-1\}.$

Prove that $*$ is commutative on $\mathbf{R} - \{-1\}$. Find the identity element and prove that every element of $\mathbf{R} - \{-1\}$ is invertible.

- **25.** Prove that the perimeter of a right angled triangle of given hypotenuse is maximum when the triangle is isosceles.
- **26.** Using the method of integration, find the area of the region bounded by the lines

Using matrix
$$
B = 2A + 3y + 4z = 17
$$
 and $y + 2z = 7$.

\nOn the set $\mathbb{R} - \{-1\}$, a binary operation is defined by $a * b = a + b + ab$ for all $a, b \in \mathbb{R} - \{-1\}$.

\ne that $*$ is commutative on $\mathbb{R} - \{-1\}$. Find the identity element of $\mathbb{R} - \{-1\}$ is invertible.

\nProve that the perimeter of a right-angled triangle of given hyp when the triangle is isosceles.

\nUsing the method of integration, find the area of the region $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$.

\nOR

\nEvaluate $\int_{1}^{4} (2x^2 - x) dx$ as limit of a sum.

Evaluate $\int (2x^2 -$ 4 2 y d $\frac{1}{2}$ $\frac{1}{2}$ 1

27. Find the co-ordinates of the foot of perpendicular from the point (2, 3, 7) to the plane $3x - y - z = 7$. Also, find the length of the perpendicular.

OR

$$
\vec{r} = \hat{i} + \hat{j} + \lambda(\hat{i} + 2\hat{j} - \hat{k})
$$
 and $\vec{r} = \hat{i} + \hat{j} + \mu(-\hat{i} + \hat{j} - 2\hat{k})$.

Also, find the distance of this plane from the point $(1,1,1)$

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Find the co-ordinates of the foot of perpendicular from the point (2, 3, 7) to the

plane 3x − y − z = 7. Also, find the length of the perpendicular.

OR

Find the equation of the pla **28.** Two cards are drawn successively without replacement from well shuffled pack of 52 cards. Find the probability distribution of the number of kings. Also, calculate the mean and variance of the distribution.
- **29.** A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contains atleast 8 units of Vitamin A and 10 units of Vitamin C. Food 'I' contains 2 units/kg of Vitamin A and 1 unit/kg of Vitamin C. Food 'II' contains 1 unit/kg of Vitamin A and 2 units/kg of Vitamin C. It costs Rs 50 per kg to purchase Food 'I' and Rs 70 per kg to purchase Food 'II'. Formulate this problem as a linear programming problem to minimise the cost of such a mixture and solve it graphically.

7. 0
\n8.
$$
x + c
$$

\n9. $\lambda = -2$
\n10. $\frac{+1}{7}$
\n1 × 10 = 10

Sections —B

11. L.H.S. = –1 1 sin 1 – sin cot 1 sin 1 – sin *x x x x* + + + − ⁼ ² ² –1 2 2 cos sin cos – sin 2 2 2 2 cot cos sin – cos – sin 2 2 2 2 *x x x x x x x x* + + + 1 1 2 cos sin cos – sin 2 2 2 2 cos sin – cos – sin 2 2 2 2 *x x x x x x x x* + + since 0 cos sin 2 4 2 2 *x x x* π < < ⇒ > cos sin cos –sin 2 2 2 2 cot *x x x x x x x x* + +

$$
= \cot^{-1}\left\{\frac{\left|\cos\frac{x}{2} + \sin\frac{x}{2}\right| + \left|\cos\frac{x}{2} - \sin\frac{x}{2}\right|}{\left|\cos\frac{x}{2} + \sin\frac{x}{2}\right| - \left|\cos\frac{x}{2} - \sin\frac{x}{2}\right|}\right\} \quad \left[\text{since } 0 < \frac{x}{2} < \frac{\pi}{4} \Rightarrow \cos\frac{x}{2} > \sin\frac{x}{2}\right]
$$

$$
= \cot^{-1}\left\{\frac{\sqrt{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2} + \sqrt{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}}{\sqrt{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2} - \sqrt{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}}\right\}
$$

\n
$$
= \cot^{-1}\left\{\frac{\left|\cos\frac{x}{2} + \sin\frac{x}{2}\right| + \left|\cos\frac{x}{2} - \sin\frac{x}{2}\right|}{\left|\cos\frac{x}{2} + \sin\frac{x}{2}\right| - \left|\cos\frac{x}{2} - \sin\frac{x}{2}\right|}\right\} \left[\text{since } 0 < \frac{x}{2} < \frac{\pi}{4} \Rightarrow \cos\frac{x}{2} > \sin\frac{x}{2}\right]
$$

\n
$$
= \cot^{-1}\left\{\frac{\cos\frac{x}{2} + \sin\frac{x}{2} + \cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2} - \cos\frac{x}{2} + \sin\frac{x}{2}}\right\}
$$

\n
$$
= \cot^{-1}\left\{\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right\} = \cot^{-1}\left\{\cot\frac{x}{2}\right\} = \frac{x}{2}
$$

$$
\left[\text{since } 0 < \frac{x}{2} < \frac{\pi}{4}\right]
$$
\n
$$
\left[\text{since } 0 < \frac{x}{2} < \frac{\pi}{4}\right]
$$
\nOR\n
$$
\sin^{-1}x + \sin^{-1}2x = \frac{\pi}{3}
$$
\n
$$
\Rightarrow \sin^{-1}2x = \frac{\pi}{3} - \sin^{-1}x
$$
\n
$$
\Rightarrow 2x = \sin(\frac{\pi}{3} - \sin^{-1}x)
$$
\n
$$
= \sin \frac{\pi}{3} \cos(\sin^{-1}x) - \cos \frac{\pi}{3} \sin(\sin^{-1}x) = \frac{\sqrt{3}}{2} \sqrt{1 - \sin^{-2}(\sin^{-1}x)} - \frac{1}{2}x
$$
\n
$$
= \frac{\sqrt{3}}{2} \sqrt{1 - x^2} - \frac{1}{2}x
$$
\n
$$
4x = \sqrt{3} \sqrt{1 - x^2} - x, 5x = \sqrt{3} \sqrt{1 - x^2}
$$
\n
$$
\Rightarrow 25x^2 = 3(1 - x^2)
$$
\n
$$
\Rightarrow 28x^2 = 3
$$
\n
$$
\Rightarrow x^2 = \frac{3}{28}
$$
\n
$$
\Rightarrow x = \pm \frac{1}{2} \sqrt{\frac{3}{7}}
$$
\n
$$
1
$$

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\nHence
$$
x = \frac{1}{2}\sqrt{\frac{3}{7}}
$$
 (as $x > 0$ given)
\n
$$
\frac{1}{2}
$$
\nThus $x = \frac{1}{2}\sqrt{\frac{3}{7}}$ is the solution of given equation.
\n12. Let $\Delta = \begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix}$
\nUsing $C_1 \rightarrow C_1 + C_2 + C_3$, we get
\n
$$
\Delta = \begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(p+q+r) & r+p & p+q \\ 2(x+y+z) & z+x & x+y \end{vmatrix}
$$
\n
$$
= 2 \begin{vmatrix} a+b+c & c+a & a+b \\ p+q+r & r+p & p+q \\ x+y+z & z+x & x+y \end{vmatrix}
$$

12. Let
$$
\Delta = \begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix}
$$

Using $C_1 \rightarrow C_1 + C_2 + C_3$, we get

MATHEMATICS
\n
$$
x = \frac{1}{2} \sqrt{\frac{3}{7}} \text{ (as } x > 0 \text{ given)}
$$
\n
$$
x = \frac{1}{2} \sqrt{\frac{3}{7}} \text{ is the solution of given equation.}
$$
\nLet $\Delta = \begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix}$
\nUsing $C_1 \rightarrow C_1 + C_2 + C_3$, we get\n
$$
\Delta = \begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(p+q+r) & r+p & p+q \\ 2(x+y+z) & z+x & x+y \end{vmatrix}
$$
\n
$$
= 2 \begin{vmatrix} a+b+c & c+a & a+b \\ p+q+r & r+p & p+q \\ x+y+z & z+x & x+y \end{vmatrix}
$$
\nUsing $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we get\n
$$
|a+b+c - b - -c|
$$

$$
=2\begin{vmatrix} a+b+c & c+a & a+b \\ p+q+r & r+p & p+q \\ x+y+z & z+x & x+y \end{vmatrix}
$$

Using $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we get

$$
|y+z z+x + y|
$$

\nUsing C₁ \rightarrow C₁+C₂+C₃, we get
\n
$$
\Delta = \begin{vmatrix}\n2(a+b+c) & c+a & a+b \\
2(p+q+r) & r+p & p+q \\
2(x+y+z) & z+x & x+y\n\end{vmatrix}
$$
\n
$$
= 2 \begin{vmatrix}\na+b+c & c+a & a+b \\
p+q+r & r+p & p+q \\
x+y+z & z+x & x+y\n\end{vmatrix}
$$
\nUsing C₂ \rightarrow C₂-C₁ and C₃ \rightarrow C₃-C₁, we get
\n
$$
\Delta = 2 \begin{vmatrix}\na+b+c & -b & -c \\
p+q+r & -q & -r \\
x+y+z & -y & -z\n\end{vmatrix}
$$
\n $1\frac{1}{2}$
\nUsing C₁ \rightarrow C₁+C₂+ C₃ and taking (-1) common from both C₂ and C₃

1

Using $C_1 \rightarrow C_1 + C_2 + C_3$ and taking (– 1) common from both C_2 and C_3

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\n
$$
\Delta = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}
$$
\nCase 1 when $x < -2$

\n
$$
f(x) = |x + 1| + |x + 2| = -(x + 1) - (x + 2) = -2x - 3
$$
\nCase 2 When $x < -2$

13. Case 1 when $x < -2$

 $f(x) = |x + 1| + |x + 2| = -(x + 1) - (x + 2) = -2x - 3$ Case 2 When $-2 \le x < -1$ $f(x) = -x - 1 + x + 2 = 1$ Case 3 When $x \ge -1$ $f(x) = x + 1 + x + 2 = 2x + 3$ b c

q r

y z

then x < -2

+ 1| + |x + 2| = - (x + 1) - (x+2) = -2x -3

Then - 2 ≤ x < -1

7| - 1 + x + 2 = 1

Then x ≥ -1

+ 1 + x + 2 = 2x + 3

-2x -3 when $x \ge -1$

1 when -2≤ x < -1

1 when -2≤ x < -1

1 x + 3 whe $=2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

ase 1 when $x < -2$
 $(x) = |x + 1| + |x + 2| = -(x + 1) - (x + 2) = -2x - 3$

ase 2 When $-2 \le x < -1$
 $x) = -x - 1 + x + 2 = 1$

ase 3 When $x \ge -1$
 $x) = x + 1 + x + 2 = 2x + 3$

aus
 $(x) =\begin{cases} -2x - 3 & \text{when } x < -2 \\ 1 & \text{when } -$ q r
 $\begin{vmatrix} 1 \\ y \\ z \end{vmatrix}$

hen $x < -2$
 $\begin{vmatrix} 1 + x + 2 & = -x + 1 \\ -1 + x + 2 & = 1 \end{vmatrix}$

hen $x \ge -1$
 $\begin{vmatrix} 1 + x + 2 & = 2x + 3 \\ 1 & \text{when } -2 \le x < -1 \end{vmatrix}$
 $\begin{vmatrix} 2x + 3 & \text{when } x \ge -1 \\ x = -2, \lim_{x \to -2^{-}} f(x) = \lim_{x \to -2} (-2x - 3) = 4 - 3 = 1$
 $\begin{vmatrix}$ $\Delta = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$
 $\text{Case 1 when } x < -2$
 $f(x) = |x + 1| + |x + 2| = -(x + 1) - (x + 2) = -2x - 3$
 $\text{Case 2 When } -2 \le x < -1$
 $f(x) = -x - 1 + x + 2 = 1$
 $f(x) = x + 1 + x + 2 = 2x + 3$

Thus
 $f(x) = \begin{cases} -2x - 3 & \text{when } x < -2 \\ 1 & \text{when } -2 \le x < -1 \\ 2x + 3 & \text{$ $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

1 when $x < -2$
 $= |x + 1| + |x + 2| = -(x + 1) - (x + 2) = -2x - 3$

2 When $-2 \le x < -1$
 $= -x - 1 + x + 2 = 1$

3 When $x \ge -1$
 $= x + 1 + x + 2 = 2x + 3$
 $= \begin{cases} -2x - 3 & \text{when } x < -2 \\ 1 & \text{when } -2 \le x < -1 \\ 2x + 3 & \text{when } x \ge -1 \end$ $\Delta = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

13. Case 1 when $x < -2$
 $f(x) = |x + 1| + |x + 2| = -(x + 1) - (x + 2) = -2x - 3$

Case 2 When $-2 \le x < -1$
 $f(x) = -x - 1 + x + 2 = 1$

Case 3 When $x \ge -1$
 $f(x) = \begin{cases} -2x - 3 & \text{when } x < -2 \\ 1 & \text{when } -2 \le x < -1 \\ 2x + 3 &$ *f* $f(x) = \lim_{x \to -2} 1 - (x+2) = -2x -3$
 f $f(x) = \lim_{x \to -2} 2x + 3$
 f $f(x) = \lim_{x \to -2} (-2x-3) = 4 - 3 = 1$
 f $f(x) = \lim_{x \to -2} 1 = 1$
 f $f(x) = \lim_{x \to -2} 1 = 1$
 f $f(x) = \lim_{x \to -2} 1 = 1$
 f $f(x) = \lim_{x \to -2} 1 = 1$ $-(x+2) = -2x-3$
 -1
 \therefore $\Delta = 2 \begin{vmatrix} p & q & r \\ x & y & z \end{vmatrix}$
 13. Case 1 when $x < -2$
 $f(x) = |x + 1| + |x + 2| = -(x + 1) - (x + 2) = -2x - 3$

Case 2 When $-2 \le x < -1$
 $f(x) = -x - 1 + x + 2 = 1$

Case 3 When $x \ge -1$
 $f(x) = x + 1 + x + 2 = 2x + 3$

Thus
 $f(x) = \begin{cases} -2x - 3 & \text{when } x <$ *f* $x + 2 = -x + 1$ → $y = -2x - 3$
 $-2 \le x < -1$
 $x + 2 = 1$
 $x \ge -1$
 $x + 2 = 2x + 3$
 $x + 2 = 2x + 3$
 $x \ge -1$
 $x \ge -1$
 $y \ge -1$
 $y \ge -1$
 $y \ge -1$
 $y \ge -2$
 + 1) - (x+2) = -2x -3
 $\frac{1}{2}$
 $f(x) = |x + 1| + |x + 2| = -(x + 1) - (x+2) = -2$

Case 2 When $-2 \le x < -1$
 $f(x) = -x - 1 + x + 2 = 1$

Case 3 When $x \ge -1$
 $f(x) = x + 1 + x + 2 = 2x + 3$

Thus

Thus
 $f(x) =\begin{cases} -2x-3 & \text{when } x < -2 \\ 1 & \text{when } -2 \le x < -1 \\ 2x+3 & \text{when } x \ge -1 \end{cases}$

Now, L.H.S at *f* $x + 1 + 1 + x + 2 = -(x + 1) - (x+2) = -2x - 3$
 $f(x) = -x - 1 + x + 2 = 1$
 $f(x) = -x - 1 + x + 2 = 1$
 $f(x) = \begin{cases} -2x - 3 & \text{when } x \ge -1 \\ 1 & \text{when } -2 \le x < -1 \\ 2x + 3 & \text{when } x \ge -1 \end{cases}$
 $f(x) = \lim_{x \to -2} f(x) = \lim_{x \to -2} (-2x - 3) = 4 - 3 = 1$
 $f(x) = -2 - 1 - 2 + 1 + 1 + 2 +$

Thus

$$
f(x) = \begin{cases} -2x-3 & \text{when } x < -2 \\ 1 & \text{when } -2 \le x < -1 \\ 2x+3 & \text{when } x \ge -1 \end{cases}
$$

 $\lim_{x \to -2^{-}} f(x) = \lim_{x \to -2^{-}} (-2x-3) = 4-3=1$

 $\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} 1 = 1$ Also $f(-2) = |-2 + 1| + |-2 + 2| = |-1| + |0| = 1$ + 3
 $x < -2$
 $-2 \le x < -1$
 $x \ge -1$
 $f(x) = \lim_{x \to -2} (-2x - 3) = 4 - 3 = 1$
 $= \lim_{x \to -2} 1 = 1$
 $2| = |-1| + |0| = 1$
 $\lim_{x \to -2} f(x)$
 $= 4 - 3 = 1$
 $1\frac{1}{2}$
 $= 8$ at $x = -2$

Thus,
$$
\lim_{x \to -2^{-}} f(x) = f(-2) = \lim_{x \to -2^{+}} f(x)
$$
 $1\frac{1}{2}$

 \Rightarrow The function *f* is continuous at $x = -2$

 $\lim_{x \to -1^{-}} f(x) = \lim_{x \to -1^{-}} 1 = 1$ $\lim_{n \to \infty} 1 = 1$

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\nNow, L.H.S at
$$
x = -1
$$
, $\lim_{x \to -1^{-}} f(x) = \lim_{x \to -1^{-}} 1 = 1$
\nR.H.S at $x = -1$, $\lim_{x \to -1^{+}} f(x)$
\n
$$
= \lim_{x \to -1^{+}} (2x+3) = 1
$$
\nAlso $f(-1) = |-1 + 1| + |-1 + 2| = 1$
\nThus, $\lim_{x \to -1^{+}} f(x) = \lim_{x \to -1^{-}} = f(-1)$
\n \Rightarrow The function is continuous at $x = -1$
\nHence, the given function is continuous at both the points $x = -1$ and $x = -2$
\n14. $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$

Also $f(-1) = |-1 + 1| + |-1 + 2| = 1$

 $\lim_{x \to -1^+} f(x) = \lim_{x \to -1^-} f(-1)$

 \Rightarrow The function is continuous at $x = -1$

Hence, the given function is continuous at both the points $x = -1$ and $x = -2$

14. $x = 2\cos\theta - \cos2\theta$ and $y = 2\sin\theta - \sin2\theta$

Now, L.H.S at
$$
x = -1
$$
, $\lim_{x \to -1^{-}} f(x) = \lim_{x \to -1^{-}} 1 = 1$
\nR.H.S at $x = -1$, $\lim_{x \to -1^{+}} f(x)$
\n
$$
= \lim_{x \to -1^{+}} (2x + 3) = 1
$$
\nAlso $f(-1) = |-1 + 1| + |-1 + 2| = 1$
\nThus, $\lim_{x \to -1^{+}} f(x) = \lim_{x \to -1^{-}} = f(-1)$
\n \Rightarrow The function is continuous at $x = -1$
\nHence, the given function is continuous at both the points $x = -1$ and $x = -2$
\n14. $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$
\nSo $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{d\theta}{d\theta}} = \frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin \theta} = \frac{-2\sin\frac{3\theta}{2}\sin\left(\frac{-\theta}{2}\right)}{2\cos\frac{3\theta}{2}\sin\frac{\theta}{2}} = \tan\frac{3\theta}{2}$
\nDifferentiating both sides w.r.t. x, we get
\n $\frac{d^2y}{dx^2} = \frac{3}{2}\sec^2\frac{3\theta}{2} \times \frac{d\theta}{dx}$

1

2

1 $1\frac{1}{2}$

 $1\frac{1}{2}$

 $1\frac{1}{2}$

Differentiating both sides w.r.t. *x*, we get

Also
$$
f(-1) = |-1 + 1| + |-1 + 2| = 1
$$

\nThus, $\lim_{x \to -1^{-}} f(x) = \lim_{x \to -1^{-}} f(-1)$
\n \Rightarrow The function is continuous at $x = -1$
\nHence, the given function is continuous at both the points $x = -1$ and $x = -2$
\n14. $x = 2\cos\theta - \cos 2\theta$ and $y = 2 \sin\theta - \sin 2\theta$
\nSo $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin \theta} = \frac{-2\sin\frac{3\theta}{2}\sin\left(\frac{-\theta}{2}\right)}{2\cos\frac{3\theta}{2}\sin\frac{\theta}{2}} = \tan\frac{3\theta}{2}$
\nDifferentiating both sides w.r.t. x, we get
\n
$$
\frac{d^2y}{dx^2} = \frac{3}{2}\sec^2\frac{3\theta}{2} \times \frac{d\theta}{dx}
$$
\n
$$
= \frac{3}{2}\sec^2\frac{3\theta}{2} \times \frac{1}{2(\sin 2\theta - \sin \theta)} = \frac{3}{4}\sec^2\frac{3\theta}{2} \times \frac{1}{2\cos\frac{3\theta}{2}\sin\frac{\theta}{2}}
$$
\n
$$
= \frac{3}{8}\sec^3\frac{3\theta}{2}\csc\frac{\theta}{2}
$$

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\nThus
$$
\frac{d^2y}{dx^2}
$$
 at $\theta = \frac{\pi}{2}$ is $\frac{3}{8} \sec^3 \frac{3\pi}{4} \csc \frac{\pi}{4} = \frac{-3}{2}$

\nOR

\nWe have

\n $x\sqrt{1+y} + y\sqrt{1+x} = 0$

\n $\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$

\nSquaring both sides, we get

\n $x^2(1+y) = y^2(1+x)$

\n $\Rightarrow (x+y)(x-y) = -yx(x-y)$

$$
\mathbf{OR} \qquad \qquad
$$

We have

Squaring both sides, we get

IDENTIFY:
$$
\frac{d^{2}y}{dx^{2}} \text{ at } \theta = \frac{\pi}{2} \text{ is } \frac{3}{8} \sec^{3} \frac{3\pi}{4} \csc \frac{\pi}{4} = \frac{-3}{2}
$$

\nOR

\nWe have

\n
$$
x\sqrt{1+y} + y\sqrt{1+x} = 0
$$
\n
$$
\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}
$$
\nSquaring both sides, we get

\n
$$
x^{2}(1+y) = y^{2}(1+x)
$$
\n
$$
\Rightarrow (x+y)(x-y) = -yx(x-y)
$$
\n
$$
\Rightarrow x+y = -xy, \text{ i.e., } y = \frac{-x}{1+x}
$$
\n
$$
\Rightarrow \frac{dy}{dx} = -\left\{\frac{(1+x) \cdot 1 - x(0+1)}{(1+x)^{2}}\right\} = \frac{-1}{(1+x)^{2}}
$$
\n15. Let OAB be a cone and let LM be the level of

\n
$$
\text{water at any time } t.
$$
\nLet ON = h and MN = r

\nGiven AB = 10 cm, OC = 10 cm and
$$
\frac{dV}{dt} = 4 cm^{3}
$$

\nNow, we have

$$
\Rightarrow \frac{dy}{dx} = -\left\{ \frac{(1+x) \cdot (1-x)(0+1)}{(1+x)^2} \right\} = \frac{-1}{(1+x)^2}
$$

15. Let OAB be a cone and let LM be the level of water at any time *t.*

Let ON =
$$
h
$$
 and MN = r

Given AB = 10 cm, OC = 10 cm and
$$
\frac{dV}{dt}
$$
 = 4 cm

minute, where V denotes the volume of cone OLM.

Note that Δ ONM ~ Δ OCB

2

1

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\n
$$
\Rightarrow \frac{MN}{CB} = \frac{ON}{OC} \text{ or } \frac{r}{5} = \frac{h}{10} \Rightarrow r = \frac{h}{2}
$$
\n1
\nNow, $V = \frac{1}{3} \pi r^2 h$ (i)
\nSubstituting $r = \frac{h}{2}$ in (i), we get

 $1\frac{1}{2}$ 2

1

2

 $1\frac{1}{2}$

Now,
$$
V = \frac{1}{3}\pi r^2 h
$$

Substituting
$$
r = \frac{h}{2}
$$
 in (i), we get

$$
V=\frac{1}{12}\pi h^3
$$

Differentiating w.r.t.*t*

318 MATHEMATICS
\n
$$
\Rightarrow \frac{MN}{CB} = \frac{ON}{OC} \text{ or } \frac{r}{5} = \frac{h}{10} \Rightarrow r = \frac{h}{2}
$$
\n1
\nNow, $V = \frac{1}{3} \pi r^2 h$ (i)
\nSubstituting $r = \frac{h}{2} \text{ in (i), we get}$
\n
$$
V = \frac{1}{12} \pi h^3
$$
\nDifferentiating w.r.t.
\n
$$
\frac{dV}{dt} = \frac{3\pi h^2}{12} \frac{dh}{dt}
$$
\n
$$
\Rightarrow \frac{dh}{dt} = \frac{4}{\pi h^2} \frac{dv}{dt}
$$
\nTherefore, when $h = 6 \text{ cm}$, $\frac{dh}{dt} = \frac{4}{9\pi} \text{ cm/minute}$
\n
$$
f(x) = x^3 + \frac{1}{x^3}
$$
\n
$$
\Rightarrow f'(x) = 3x^3 - \frac{3}{x^4}
$$
\n
$$
= \frac{3}{x^4} \left(\frac{x^6 - 1}{x^4}\right) = \frac{3}{x^4} \left(\frac{x^2 - 1}{x^4}\right) \left(\frac{x^4 + x^2 + 1}{x^4}\right)
$$

1 $1\frac{1}{2}$ 2

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\nAs
$$
x^4 + x^2 + 1 > 0
$$
 and $x^4 > 0$, therefore, for *f* to be increasing, we have

\n $x^2 - 1 > 0$

\n $\Rightarrow x \in (-\infty, -1) \cup (1, \infty)$

\n $\Rightarrow 1\frac{1}{2}$

\nThus *f* is increasing in $(-\infty, -1) \cup (1, \infty)$

\n(ii) For *f* to be decreasing $f'(x) < 0$

\n $\Rightarrow x^2 - 1 < 0$

\n $\Rightarrow (x - 1)(x + 1) < 0 \Rightarrow x \in (-1, 0) \cup (0, 1)$ $[x \neq 0 \text{ as } f \text{ is not defined at } x = 0]$ $1\frac{1}{2}$

$$
x^2 - 1 > 0
$$

$$
\Rightarrow x \in (-\infty, -1) \cup (1, \infty)
$$

(ii) For *f* to be decreasing $f'(x) < 0$

$$
\Rightarrow x^2 - 1 < 0
$$

$$
\Rightarrow (x-1)(x+1) < 0 \Rightarrow x \in (-1, 0) \cup (0, 1) \ [x \neq 0 \text{ as } f \text{ is not defined at } x = 0] \ 1\frac{1}{2}
$$

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\n319

\nAs
$$
x^4 + x^2 + 1 > 0
$$
 and $x^4 > 0$, therefore, for *f* to be increasing, we have

\n $x^2 - 1 > 0$

\n⇒ $x \in (-\infty, -1) \cup (1, \infty)$

\nThus *f* is increasing in $(-\infty, -1) \cup (1, \infty)$

\n(ii) For *f* to be decreasing $f'(x) < 0$

\n⇒ $x^2 - 1 < 0$

\n⇒ $x^2 - 1 < 0$

\n⇒ $(x - 1)(x + 1) < 0 \Rightarrow x \in (-1, 0) \cup (0, 1)$ $[x \neq 0 \text{ as } f \text{ is not defined at } x = 0]$ $1\frac{1}{2}$

\nThus $f(x)$ is decreasing in $(-1, 0) \cup (0, 1)$

\n16. Let
$$
\frac{3x - 2}{(x + 3)(x + 1)^2} = \frac{A}{x + 3} + \frac{B}{x + 1} + \frac{C}{(x + 1)^2}
$$

\nThen $3x - 2 = A(x + 1)^2 + B(x + 1)(x + 3) + C(x + 3)$

\ncomparing the coefficient of x^2 , x and constant, we get

\nA + B = 0, 2A + 4B + C = 3 and A + 3B + 3C = -2

\nSolving these equations, we get

Then $3x - 2 = A(x + 1)^2 + B(x + 1)(x + 3) + C(x + 3)$ comparing the coefficient of x^2 , x and constant, we get

 $A + B = 0$, $2A + 4B + C = 3$ and $A + 3B + 3C = -2$

Solving these equations, we get

(ii) For *f* to be decreasing *f*′(*x*) < 0
\n⇒
$$
x^2 - 1 < 0
$$

\n⇒ $(x - 1) (x + 1) < 0 \Rightarrow xe (-1, 0) \cup (0, 1) [x ≠ 0 \text{ as } f \text{ is not defined at } x = 0]$ $\frac{1}{2}$
\nThus *f*(*x*) is decreasing in (-1, 0) ∪ (0, 1)
\n16. Let $\frac{3x - 2}{(x + 3)(x + 1)^2} = \frac{A}{x + 3} + \frac{B}{x + 1} + \frac{C}{(x + 1)^2}$
\nThen $3x - 2 = A(x + 1)^2 + B(x + 1) (x + 3) + C(x + 3)$
\ncomparing the coefficient of *x*², *x* and constant, we get
\n $A + B = 0, 2A + 4B + C = 3$ and $A + 3B + 3C = -2$
\nSolving these equations, we get
\n
$$
A = \frac{-11}{4}, B = \frac{11}{4} \text{ and } C = \frac{-5}{2}
$$
\n
$$
\Rightarrow \frac{3x - 2}{(x + 3)(x + 1)^2} = \frac{-11}{4(x + 3)} + \frac{11}{4(x + 1)} - \frac{5}{2(x + 1)^2}
$$

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\nHence
$$
\int \frac{3x-2}{(x+3)(x+1)^2} dx = \frac{-11}{4} \int \frac{1}{x+3} dx + \frac{11}{4} \int \frac{1}{x+1} dx - \frac{5}{2} \int \frac{1}{(x+1)^2} dx
$$
\n
$$
= \frac{-11}{4} \log|x+3| + \frac{11}{4} \log|x+1| + \frac{5}{2(x+1)} + C_1
$$
\nOR
\n
$$
\int \left[\log(\log x) + \frac{1}{(\log x)^2} \right] dx
$$
\n
$$
= \int \log(\log x) dx + \int \frac{1}{(\log x)^2} dx
$$
\nIntegrating log (log x) by parts, we get
\n
$$
\int \log(\log x) dx = x \log(\log x) - \int \frac{x}{(\log x)} \times \frac{1}{x} dx
$$
\n
$$
= x \log(\log x) - \int \frac{1}{\log x} dx
$$
\n
$$
= x \log(\log x) - \int \frac{1}{\log x} dx
$$
\n
$$
= x \log(\log x) - \int \frac{1}{\log x} dx
$$
\n
$$
= x \log(\log x) - \frac{x}{\log x} - \int \frac{1}{(\log x)^2} dx
$$

$$
= x \log (\log x) - \left[\frac{x}{\log x} - \int x \left(\frac{-1}{(\log x)^2} \right) \times \frac{1}{x} dx \right]
$$

$$
= x \log (\log x) - \frac{x}{\log x} - \int \frac{1}{(\log x)^2} dx
$$

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\n321

\nTherefore,
$$
\int \left(\log(\log x) + \frac{1}{(\log x)^2} \right) dx = x \log(\log x) - \frac{x}{\log x} + C
$$

\n11

\n12

\n13

\n14

\n15

\n16

\n17. Let I =
$$
= \int_{0}^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx
$$

$$
\begin{aligned}\n\text{DESIGN OF THE QUESTOR PAPER} \quad & 321 \\
\text{Therefore, } & \int \log(\log x) + \frac{1}{(\log x)^2} \, dx = x \log(\log x) - \frac{x}{\log x} + C & \quad 1\frac{1}{2} \\
17. \quad \text{Let } I = \frac{5}{\pi} \int_0^x \frac{x \sin x}{1 + \cos^2 x} \, dx \\
& = \frac{5}{\pi} \left(\frac{\pi - x}{1 + \cos^2 x} \right) dx \quad \left[\text{since } \int_0^a \int x \, dx \right]_0^a \int dx \\
& = \int_0^x \frac{\pi \sin x}{1 + \cos^2 x} \, dx - I \\
\Rightarrow & 2I = \pi \int_0^x \frac{\sin x}{1 + \cos^2 x} \, dx \\
\text{Put } \cos x = t \text{ for } x = \pi \Rightarrow t = -1, \ x = 0 \Rightarrow t = +1 \text{ and } -\sin x \, dx = dt. \\
\text{Therefore } 2I = \pi \int_0^1 \frac{-dt}{1 + t^2} \quad & \int_0^1 \frac{dt}{1 + t^2} \\
& = \pi \left[\tan^{-1} t \right]_{-1}^1 = \pi \left[\tan^{-1} (+1) - \tan^{-1} (-1) \right] \\
& = + \pi \left[\frac{\pi}{2} \right] = \frac{\pi^2}{2}\n\end{aligned}
$$

Therefore
$$
2I = \pi \int_{1}^{-1} \frac{-dt}{1+t^2} = \pi \int_{-1}^{1} \frac{dt}{1+t^2}
$$
 $1\frac{1}{2}$

$$
= \int_{0}^{\pi} \frac{\pi \sin x}{1 + \cos^{2} x} dx - I
$$

\n
$$
\Rightarrow 2I = \pi \int_{0}^{\pi} \frac{\sin x}{1 + \cos^{2} x} dx
$$

\nPut $\cos x = t$ for $x = \pi \Rightarrow t = -1$, $x = 0 \Rightarrow t = +1$ and $-\sin x dx = dt$.
\nTherefore $2I = \pi \int_{1}^{\frac{\pi}{4}} \frac{-dt}{1 + t^{2}} = \pi \int_{-1}^{1} \frac{dt}{1 + t^{2}}$
\n
$$
= \pi \left[\tan^{-1} t \right]_{-1}^{1} = \pi \left[\tan^{-1} (+1) - \tan^{-1} (-1) \right]
$$

\n
$$
= + \pi \left[\frac{\pi}{2} \right] = \frac{\pi^{2}}{2}
$$

\n
$$
I = \frac{\pi^{2}}{4}
$$

\n
$$
I = \frac{\pi^{2}}{4}
$$

18. The equation of circles which pass through the origin and whose centre lies on $x - axis$ is MATHEMATICS

the equation of circles which pass through the origin and wt
 $-$ axis is
 $x-a)^2 + y^2 = a^2$... (i)

differentiating *w.r.t.x*, we get
 $(x-a)+2y\frac{dy}{dx} = 0$ MATHEMATICS

The equation of circles which pass through the origin and
 $(x-a)^2 + y^2 = a^2$ (i)

Differentiating w.r.t.x, we get
 $2(x-a)+2y\frac{dy}{dx} = 0$
 $\Rightarrow x+y\frac{dy}{dx} = a$

Substituting the value of a in (i), we get
 $\left(y\frac{dy}{dx$ MATHEMATICS

e equation of circles which pass through the ori

axis is
 $-a)^2 + y^2 = a^2$

fferentiating *w.r.t.x*, we get
 $x-a)+2y\frac{dy}{dx} = 0$
 $x + y\frac{dy}{dx} = a$

bstituting the value of *a* in (i), we get MATICS

tion of circles which pass through the origin and w

s
 $+ y^2 = a^2$ (i)

iating *w.r.t.x*, we get
 $+ 2y \frac{dy}{dx} = 0$
 $\frac{dy}{dx} = a$

ing the value of *a* in (i), we get MATHEMATICS
 x equation of circles which pass through the origin

axis is
 $(a-a)^2 + y^2 = a^2$
 x $(a-a)^2 + y^2 = a^2$
 x $x-a) + 2y \frac{dy}{dx} = 0$
 $x + y \frac{dy}{dx} = a$

sostituting the value of *a* in (i), we get
 $\left(\frac{dy}{dx}\right)^2 + y^2$ MATHEMATICS

The equation of circles which pass through the origin a
 $(x-a)^2 + y^2 = a^2$ (i)

Differentiating w.r.t.x, we get
 $2(x-a)+2y\frac{dy}{dx}=0$
 $\Rightarrow x+y\frac{dy}{dx}=a$

Substituting the value of a in (i), we get
 $\left(y\frac{dy}{dx}\right)^2 +$ The equation of circles which pass through the origin and whose centre lies on
 $x - ax$ is is
 $(x-a)^2 + y^2 = a^2$... (i) $1\frac{1}{2}$

Differentiating w.r.t.x, we get
 $2(x-a)+2y\frac{dy}{dx}=0$
 $\Rightarrow x+y\frac{dy}{dx}=a$

Substituting the value of

$$
(x-a)^2 + y^2 = a^2
$$
 ... (i) $1\frac{1}{2}$

Differentiating *w.r.t.x*, we get

MATHEMATICS
\nThe equation of circles which pass through t
\n
$$
(x-a)^2 + y^2 = a^2
$$
\nDifferentiating w.r.t.x, we get
\n
$$
2(x-a)+2y\frac{dy}{dx} = 0
$$
\n
$$
\Rightarrow x+y\frac{dy}{dx} = a
$$

Substituting the value of *a* in (i), we get

MATHEMATICS
\nThe equation of circles which pass through the origin and wh
\n
$$
x
$$
 – axis is
\n $(x-a)^2 + y^2 = a^2$... (i)
\nDifferentiating w.r.t.x, we get
\n $2(x-a)+2y\frac{dy}{dx} = 0$
\n $\Rightarrow x+y\frac{dy}{dx} = a$
\nSubstituting the value of *a* in (i), we get
\n $\left(y\frac{dy}{dx}\right)^2 + y^2 = \left(x+y\frac{dy}{dx}\right)^2$
\n $\Rightarrow \left(x^2 - y^2\right) + 2xy\frac{dy}{dx} = 0$
\nThe given differential equation is
\n $x^2 y dx - \left(x^3 + y^3\right) dy = 0$

19. The given differential equation is

MATHEMATICS
\nThe equation of circles which pass through the origin and whose centre lies on
\n
$$
x -
$$
 axis is
\n $(x-a)^2 + y^2 = a^2$...(i)
\nDifferentiating w.r.t.x, we get
\n $2(x-a)+2y\frac{dy}{dx}=0$
\n $\Rightarrow x+y\frac{dy}{dx} = a$
\nSubstituting the value of a in (i), we get
\n $\left(y\frac{dy}{dx}\right)^2 + y^2 = \left(x+y\frac{dy}{dx}\right)^2$
\n $\Rightarrow \left(x^2-y^2\right)+2xy\frac{dy}{dx}=0$
\nThe given differential equation is
\n $x^2y dx - \left(x^2+y^2\right) dy=0$
\n $\Rightarrow \frac{dy}{dx} = \frac{x^2y}{x^3+y^3}$...(1)
\nPut $y = vx$ so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$
\n $v + x\frac{dv}{dx} = \frac{vx^3}{x^3+v^3}$

 $1/$ $1\frac{1}{2}$ 2

$$
v + x\frac{dv}{dx} = \frac{vx^3}{x^3 + v^3x^3}
$$

$$
\Rightarrow v + x \frac{dv}{dx} = \frac{v}{1 + v^3}
$$

\n
$$
\Rightarrow x \frac{dv}{dx} = \frac{-v^4}{1 + v^3}
$$

\n
$$
\Rightarrow \int \frac{1 + v^3}{v^4} dv = -\int \frac{dx}{x}
$$

\n
$$
\Rightarrow \int \frac{1}{v^4} dv + \int \frac{1}{v} dv = -\int \frac{dx}{x}
$$

\n
$$
\Rightarrow \frac{-1}{3v^3} + \log|v| = -\log|x| + c
$$

\n
$$
\Rightarrow \frac{-x^3}{3y^3} + \log|v| = c
$$
, which is the reqd. solution.
\nWe have
\n
$$
\vec{a} \times \vec{b} = \vec{a} \times \vec{c}
$$

\n
$$
\Rightarrow \vec{a} \times \vec{b} - \vec{a} \times \vec{c} = \vec{0}
$$

\n
$$
\Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = \vec{0}
$$

20. We have

$$
\Rightarrow v + x \frac{dv}{dx} = \frac{v}{1+v^3}
$$

\n
$$
\Rightarrow x \frac{dv}{dx} = \frac{-v^4}{1+v^3}
$$

\n
$$
\Rightarrow \int \frac{1+v^3}{v^4} dv = -\int \frac{dx}{x}
$$

\n
$$
\Rightarrow \int \frac{1}{v^4} dv + \int \frac{1}{v} dv = -\int \frac{dx}{x}
$$

\n
$$
\Rightarrow \frac{-1}{3v^3} + \log |v| = -\log |x| + c
$$

\n
$$
\Rightarrow \frac{-x^3}{3y^3} + \log |v| = c
$$
, which is the read, solution.
\nWe have
\n
$$
\vec{a} \times \vec{b} = \vec{a} \times \vec{c}
$$

\n
$$
\Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = 0
$$

\n
$$
\Rightarrow \vec{a} = \vec{0} \text{ or } \vec{b} - \vec{c} = \vec{0} \text{ or } \vec{a} | (\vec{b} - \vec{c})
$$

\n
$$
\Rightarrow \vec{a} = \vec{0} \text{ or } \vec{b} - \vec{c} = \vec{b} \text{ or } \vec{a} | (\vec{b} - \vec{c})
$$

\n
$$
\Rightarrow \vec{b} - \vec{c} = \lambda \vec{a}, \text{ for some scalar } \lambda
$$

21. We know that the shorest distance between the lines $\vec{r} = \vec{a} + \lambda \vec{b}$ and $\vec{r} = \vec{c} + \mu \vec{d}$ is given by
 $D = \frac{(\vec{c} - \vec{a}) \cdot (\vec{b} \times \vec{d})}{|\vec{b} \times \vec{d}|}$ $\vec{r} = \vec{a} + \lambda \vec{b}$ and $\vec{r} = \vec{c} + \mu \vec{d}$ is given by shorest distance between the lines $\vec{r} = \vec{a} + \lambda \vec{b}$ and $\vec{r} = \vec{c} + \mu \vec{d}$ is
 $D = \frac{|\vec{c} - \vec{a}\rangle \cdot (\vec{b} \times \vec{d})|}{|\vec{b} \times \vec{d}|}$

(ons can be written as
 $(\hat{i} + \hat{j} - \hat{k})$ and $r = (\hat{i} - \hat{j} + 2\hat{k}) + \mu (-\hat{i} + 2\hat{j} + \hat{k$ est distance between the lines $\vec{r} = \vec{a} + \lambda \vec{b}$ and $\vec{r} = \vec{c} + \frac{\vec{c} - \vec{a} \cdot (\vec{b} \times \vec{d})}{|\vec{b} \times \vec{d}|}$
an be written as distance between the lines $\vec{r} = \vec{a} + \lambda \vec{b}$ and $\vec{r} = \vec{a} + \lambda \vec{b}$ and $\vec{b} \times \vec{d}$
 $\vec{b} \times \vec{d}$ \vec{b}
 $\vec{b} \times \vec{d}$ \vec{d} \vec{b}
 $\vec{b} \times \vec{d}$ \vec{c} \vec{d} \vec{b} \vec{c} \vec{d} \vec{b} \vec{d} tance between the lines $\vec{r} = \vec{a} + \lambda \vec{b}$ and $\vec{r} = \vec{c} + \mu \vec{c}$
 $\cdot (\vec{b} \times \vec{d})$

written as est distance between the lines $\vec{r} = \vec{a} + \lambda \vec{b}$ and $\vec{r} = \vec{c} + \mu \vec{d}$ is
 $\frac{\vec{c} - \vec{a} \cdot (\vec{b} \times \vec{d})}{|\vec{b} \times \vec{d}|}$

an be written as
 $\hat{i} - \hat{k}$ and $r = (\hat{i} - \hat{j} + 2\hat{k}) + \mu (-\hat{i} + 2\hat{j} + \hat{k})$ istance between the lines $\vec{r} = \vec{a} + \lambda \vec{b}$ and $\vec{r} = \vec{c} + \mu \vec{d}$ is
 $\left. \vec{a} \times \vec{a} \right|$

we written as

and $r = (\hat{i} - \hat{j} + 2\hat{k}) + \mu \left(-\hat{i} + 2\hat{j} + \hat{k} \right)$ MATHEMATICS

We know that the shorest distance between the lines $\vec{r} = \vec{a} + \lambda \vec{b}$ a

given by
 $D = \frac{|\vec{c} - \vec{a} \cdot (\vec{b} \times \vec{d})|}{|\vec{b} \times \vec{a}|}$

Now given equations can be written as
 $\vec{r} = (-\hat{i} + \hat{j} - \hat{k}) + \lambda (\hat{i} + \hat{j$ $\begin{aligned}\n\text{Let } \mathbf{r} \text{ be the three times } \vec{r} = \vec{a} + \lambda \vec{b} \text{ and } \vec{r} = \vec{c} + \mu \vec{d} \text{ is} \\
\text{Let } \mathbf{r} \text{ is } \\
r = (\hat{i} - \hat{j} + 2\hat{k}) + \mu \left(-\hat{i} + 2\hat{j} + \hat{k} \right) \\
\text{Let } \mathbf{r} \text{ is } \\
r = \left(\hat{i} - \hat{j} + 2\hat{k} \right) + \mu \left(-\hat{i} + 2\hat{j} + \hat{k} \right) \\
\text{Let } \mathbf{r} \text{ is }$ MATHEMATICS

We know that the shorest distance between the lines $\vec{r} = \vec{a} + \lambda \vec{b}$ and $\vec{r} = \vec{c} + \mu \vec{d}$ is
 $D = \left| \frac{(\vec{c} - \vec{a}) \cdot (\vec{b} \times \vec{d})}{|\vec{b} \times \vec{d}|} \right|$

Now given equations can be written as
 $\vec{r} = (-\hat{i$ s

the shorest distance between the lines $\vec{r} = \vec{a}$
 $D = \left| \frac{(\vec{c} - \vec{a}) \cdot (\vec{b} \times \vec{d})}{|\vec{b} \times \vec{d}|} \right|$

uations can be written as
 $+ \lambda (\hat{i} + \hat{j} - \hat{k})$ and $r = (\hat{i} - \hat{j} + 2\hat{k}) + \mu (-\hat{i} + \vec{a})$
 $\vec{a} = 2\hat{i} - 2\hat{j} + 3$ the shorest distance between the lines $\vec{r} = \vec{a} + \lambda \vec{b}$ and $\vec{r} = \vec{c} + \mu \vec{d}$ is
 $D = \frac{|\vec{c} - \vec{a}| \cdot (\vec{b} \times \vec{d})|}{|\vec{b} \times \vec{d}|}$

attions can be written as
 $+\lambda (\hat{i} + \hat{j} - \hat{k})$ and $r = (\hat{i} - \hat{j} + 2\hat{k}) + \mu (\hat{-i} + 2\$

$$
D = \frac{\left| \left(\vec{c} - \vec{a} \right) \cdot \left(\vec{b} \times \vec{d} \right) \right|}{\left| \vec{b} \times \vec{d} \right|}
$$

Now given equations can be written as

$$
\vec{r} = \left(-\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(\hat{i} + \hat{j} - \hat{k}\right) \text{ and } r = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \mu\left(-\hat{i} + 2\hat{j} + \hat{k}\right)
$$

and
$$
\vec{b} \times \vec{d} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix} = 3\vec{i} - 0.\vec{j} + 3\vec{k}
$$

We know that the shortest distance between the lines
$$
\vec{r} = \vec{a} + \lambda \vec{b}
$$
 and $\vec{r} = \vec{c} + \mu \vec{d}$ is given by
\n
$$
D = \frac{|\vec{c} - \vec{a}| \cdot (\vec{b} \times \vec{d})|}{|\vec{b} \times \vec{d}|}
$$
\nNow given equations can be written as
\n
$$
\vec{r} = (-\hat{i} + \hat{j} - \hat{k}) + \lambda (\hat{i} + \hat{j} - \hat{k}) \text{ and } r = (\hat{i} - \hat{j} + 2\hat{k}) + \mu (-\hat{i} + 2\hat{j} + \hat{k})
$$
\nTherefore $\vec{c} - \vec{a} = 2\hat{i} - 2\hat{j} + 3\hat{k}$
\nand $\vec{b} \times \vec{d} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix} = 3\vec{i} - 0.\vec{j} + 3\vec{k}$
\n $\Rightarrow |\vec{b} \times \vec{d}| = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$
\n $\frac{1}{2}$

 $\frac{1}{2}$

MATHEMATICS

\nWe know that the shortest distance between the lines
$$
\vec{r} = \vec{a} + \lambda \vec{b}
$$
 and $\vec{r} = \vec{c} + \mu \vec{d}$ is given by

\n
$$
D = \left| \frac{(\vec{c} - \vec{a}) \cdot (\vec{b} \times \vec{d})}{|\vec{b} \times \vec{d}|} \right|
$$
\nNow given equations can be written as

\n
$$
\vec{r} = (-\hat{i} + \hat{j} - \hat{k}) + \lambda (\hat{i} + \hat{j} - \hat{k}) \text{ and } r = (\hat{i} - \hat{j} + 2\hat{k}) + \mu (-\hat{i} + 2\hat{j} + \hat{k})
$$
\nTherefore

\n
$$
\vec{c} - \vec{a} = 2\hat{i} - 2\hat{j} + 3\hat{k}
$$
\nand

\n
$$
\vec{b} \times \vec{d} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix} = 3\hat{i} - 0\hat{j} + 3\hat{k}
$$
\nHence

\n
$$
D = \begin{vmatrix} (\vec{c} - \vec{a}) \cdot (\vec{b} \times \vec{d}) \\ |\vec{b} \times \vec{d}| \end{vmatrix} = |\vec{b} - \vec{a} + \vec{b}| = 3\sqrt{2} \Rightarrow \frac{5}{\sqrt{2}} = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}.
$$
\nHence

\n
$$
D = \begin{vmatrix} (\vec{c} - \vec{a}) \cdot (\vec{b} \times \vec{d}) \\ |\vec{b} \times \vec{d}| \end{vmatrix} = |\vec{a} - \vec{b} + \vec{c}| = \frac{5}{3\sqrt{2}} = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}.
$$
\nLet E, E, E_x, E_x, E_x, E₄ and A be the events defined as follows:

\n
$$
E_1 = \text{the missing card is a planet card.}
$$
\n
$$
E_2 = \text{the missing card is a cloud card.}
$$

22. Let E , E ₂, E ₃, E ₄ and A be the events defined as follows :

 E_1 = the missing card is a heart card,

 E_2 = the missing card is a spade card,

 E_3 = the missing card is a club card,

E_4 = the missing card is a diamond card $\frac{1}{2}$

 $A =$ Drawing two heart cards from the remaining cards.

Then
$$
P(E_1) = \frac{13}{52} = \frac{1}{4}
$$
, $P(E_2) = \frac{13}{52} = \frac{1}{4}$, $P(E_3) = \frac{13}{52} = \frac{1}{4}$, $P(E_4) = \frac{13}{52} = \frac{1}{4}$

DESIGN OF THE QUESTION PAPER 325
 E_4 = the missing card is a diamond card
 $A = \text{Drawing two heart cards from the remaining cards.}$

Then $P(E_1) = \frac{13}{52} = \frac{1}{4}$, $P(E_2) = \frac{13}{52} = \frac{1}{4}$, $P(E_4) = \frac{13}{52} = \frac{1}{4}$
 $P(A/E_1) = \text{Probability of drawing two heart cards given that one heart card is}$
 $\text{missing} = \frac{^{$ DESIGN OF THE QUESTION PAPER 325

the missing card is a diamond card
 $\frac{1}{2}$
 $\frac{13}{2} - \frac{1}{4}$, $P(E_2) = \frac{13}{52} = \frac{1}{4}$, $P(E_3) = \frac{13}{52} = \frac{1}{4}$, $P(E_4) = \frac{13}{52} = \frac{1}{4}$
 $\frac{13}{52} - \frac{1}{4}$
 $\frac{12}{52} - \frac{13}{5$ DESIGN OF THE QUESTION PAPER 325

Sing card is a diamond card
 $\frac{1}{2}$ two heart cards from the remaining cards.
 $=\frac{13}{52} = \frac{1}{4}$, $P(E_2) = \frac{13}{52} = \frac{1}{4}$, $P(E_3) = \frac{13}{52} = \frac{1}{4}$, $P(E_4) = \frac{13}{52} = \frac{1}{4}$
 \frac $P(A/E_1)$ = Probability of drawing two heart cards given that one heart card is missing = $\frac{2}{51}$ $12 \, \text{C}$ 2 51_C 2 C_2 C_2 besicos or run otestrios parent 325
 E_4 = the missing card is a diamond card
 $A = \text{Drawing two heart cards from the remaining cards.}$

Then $P(E_1) = \frac{13}{52} = \frac{1}{4}$, $P(E_2) = \frac{13}{52} = \frac{1}{4}$, $P(E_3) = \frac{13}{52} = \frac{1}{4}$, $P(E_4) = \frac{13}{52} = \frac{1}{4}$
 $P(A/E_1$ wing two heart cards given that one heart card is

wing two heart cards given that one spade card is
 $\frac{^{13}C_2}{^{51}C_2}$ and P(A/E₄) = $\frac{^{13}C_2}{^{51}C_2}$

he

(A)

(A)

P(E₁) P(A/E₁)

P(E₂) P(A/E₃) +P(E₄

 $P(A/E₂)$ = Probability of drawing two heart cards given that one spade card is missing = $\frac{2}{51}$ 13Ω $\overline{2}$ 51_C $2 \left(\frac{1}{2} \right)$ C_2 and C_3 and C_4 and C_5 and C_6 and C_7 and C_8 and C_9 and \mathbf{C}_2 and \mathbf{C}_3 and \mathbf{C}_4 and \mathbf{C}_5 and \mathbf{C}_6 and \mathbf{C}_7 and \mathbf{C}_8 and \mathbf{C}_9 and \mathbf{C}_9 and \mathbf{C}_8 and \mathbf{C}_9 and \mathbf{C}_9 and \mathbf{C}_9 and \mathbf{C}_9 and \mathbf{C}_9 and \mathbf{C}_9 a

Similarly, we have P $(A/E_3) = \frac{2}{51}$ and $^{13}C_2$ $^{13}C_2$ $^{51}C_2$ and P (A/E₄) = $^{51}C_2$ C_2 ${}^{13}C_2$ $\frac{1}{C_2}$ and P (A/E₄) = $\frac{1}{51C_2}$ 1 13_C $\overline{2}$ $\overline{1}$ 51_C 1 2 C_2 C_2 and C_1

1

By Baye's thereon, we have the

Similarly, we have
$$
P(A/E_3) = \frac{^{13}C_2}{^{51}C_2}
$$
 and $P(A/E_4) = \frac{^{13}C_2}{^{51}C_2}$

\nBy Baye's thereon, we have the required Probability = $P(E_1/A)$

\n
$$
= \frac{P(E_1) P(A/E_1)}{P(E_1) P(A/E_1) + P(E_2) P(A/E_2) + P(E_3) P(A/E_3) + P(E_4) P(A/E_4)}
$$
\n
$$
= \frac{1}{4} \frac{^{12}C_2}{^{51}C_2}
$$

E₄ = the missing card is a diamond card
\nA = Drawing two heart cards from the remaining cards.
\nThen P(E₁) =
$$
\frac{13}{52} = \frac{1}{4}
$$
, P(E₂) = $\frac{13}{52} = \frac{1}{4}$, P(E₃) = $\frac{13}{52} = \frac{1}{4}$, P(E₄) = $\frac{13}{52} = \frac{1}{4}$ V₂
\nP(A/E₁) = Probability of drawing two heart cards given that one heart card is
\nmissing = $\frac{^{12}C_2}{^{51}C_2}$
\nP(A/E₂) = Probability of drawing two heart cards given that one spade card is
\nmissing = $\frac{^{13}C_2}{^{51}C_2}$
\nSimilarly, we have P(A/E₁) = $\frac{^{13}C_2}{^{51}C_2}$ and P(A/E₄) = $\frac{^{13}C_2}{^{51}C_2}$
\n5Similarly, we have P(A/E₁) = $\frac{^{13}C_2}{^{51}C_2}$ and P(A/E₄) = $\frac{^{13}C_2}{^{51}C_2}$
\nBy Baye's theorem, we have the
\nrequired Probability = P (E₁/A)
\n= $\frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_2)P(A/E_3)P(A/E_3)P(A/E_4)P(A/E_4)}$ 1
\n= $\frac{1}{4} \times \frac{^{12}C_2}{^{51}C_2} + \frac{1}{4} \times \frac{^{13}C_2}{^{51}C_2} + \frac{1}{4} \times \frac{^{13}C_2}{^{51}C_2} + \frac{1}{4} \times \frac{^{13}C_2}{^{51}C_2}$ 1
\n= $\frac{^{12}C_2}{^{12}C_2 + ^{13}C_2 + ^{13}C_2 + ^{13}C_2} + \frac{^{13}C_2}{^{51}C_2} + \frac{1}{4} \times \frac{^{13}C_2}{^{51}C_2}$

$$
= \frac{{}^{12}C_2}{{}^{12}C_2 + {}^{13}C_2 + {}^{13}C_2 + {}^{13}C_2} = \frac{66}{66 + 78 + 78 + 78} = \frac{11}{50}
$$

Section C

1

23. We have

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\nSection C
\n23. We have
\n
$$
AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}
$$
\n
$$
= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 6I
$$
\nSimilarly BA = 6I, Hence AB = 6I = BA
\nAs AB = 6I, A⁻¹(AB) = 6A⁻¹I. This gives
\n
$$
AB = 6A^{-1}, i.e., A^{-1} = \frac{1}{6}B = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}
$$
\nThe given system of equations can be written as
\n
$$
AX = C
$$
, where
\n
$$
X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, C = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}
$$
\nThe solution of the given system $AX = C$ is given by $X = A^{-1}C$

Similarly $BA = 6I$, Hence $AB = 6I = BA$

As AB = 6I,
$$
A^{-1}(AB) = 6A^{-1}I
$$
. This gives

Section C
\nWe have
\n
$$
\begin{bmatrix}\n1 & -1 & 0 \\
2 & 3 & 4 \\
0 & 1 & 2\n\end{bmatrix}\n\begin{bmatrix}\n2 & 2 & -4 \\
4 & 2 & -4 \\
2 & -1 & 5\n\end{bmatrix}
$$
\n
$$
=\n\begin{bmatrix}\n6 & 0 & 0 \\
0 & 6 & 0 \\
0 & 0 & 6\n\end{bmatrix}\n= 6\n\begin{bmatrix}\n1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1\n\end{bmatrix}\n= 6I
$$
\nSimilarly BA = 6I, Hence AB = 6I = BA
\nAs AB = 6I, A⁻¹(AB) = 6A⁻¹I. This gives
\n
$$
11B = 6A^{-1} \text{ i.e., } A^{-1} = \frac{1}{6}B = \frac{1}{6}\n\begin{bmatrix}\n2 & 2 & -4 \\
-4 & 2 & -4 \\
2 & -1 & 5\n\end{bmatrix}
$$
\n
$$
1\frac{1}{2}
$$
\nThe given system of equations can be written as
\nAX = C, where
\n
$$
X = \begin{bmatrix}\nx \\
y\n\end{bmatrix}, C = \begin{bmatrix}\n3 \\
17\n\end{bmatrix}
$$
\nSolution of the given system AX = C is given by X = A⁻¹C
\n
$$
\frac{1}{2}
$$

The given system of equations can be written as

$$
AX = C, where
$$

$$
X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, C = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}
$$

The solution of the given system $AX = C$ is given by $X = A^{-1}C$ 1 $\frac{1}{2}$

$$
\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}
$$

$$
= \frac{1}{6} \begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+34 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}
$$

Hence $x = 2$, $y = 1$ and $z = 4$

24. Commutative: For any $a, b \in \mathbb{R} - \{-1\}$, we have $a * b = a + b + ab$ and $b * a = b + a + ba$. But {by commutative property of addition and multiplication on $R - \{-1\}$, we have:

$$
a+b+ab = b+a+ba.
$$

$$
\Rightarrow a*b = b*a
$$

Hence $*$ is commutative on $\mathbf{R} - \{-1\}$ 2

Identity Element : Let *e* be the identity element.

Then
$$
a * e = e * a
$$
 for all $a \in \mathbb{R} - \{-1\}$

$$
\Rightarrow
$$
 $a + e + ae = a$ and $e + a + ea = a$

$$
\Rightarrow e (1+a) = 0 \Rightarrow e = 0 \text{ [since } a \neq -1)
$$

Thus, 0 is the identity element for $*$ defined on $\mathbf{R} - \{-1\}$ 2

Inverse : Let $a \in \mathbb{R} - \{-1\}$ and let *b* be the inverse of *a*. Then

$$
a * b = e = b * a
$$

\n
$$
\Rightarrow a * b = 0 = b * a \quad (\because e = 0)
$$

\n
$$
\Rightarrow a + b + ab = 0
$$

$$
\Rightarrow b = \frac{-a}{a+1} \in \mathbf{R} \text{ (since } a \neq -1\text{)}
$$

MATHEMATICS
\n
$$
\Rightarrow b = \frac{-a}{a+1} \in \mathbf{R} \text{ (since } a \neq -1\text{)}
$$
\nMoreover,
$$
\frac{-a}{a+1} \neq -1. \text{Thus } b = \frac{-a}{a+1} \in \mathbf{R} - \{-1\}.
$$
\nHence, every element of $\mathbf{R} - \{-1\}$ is invertible and

Hence, every element of $\mathbf{R} - \{-1\}$ is invertible and

the inverse of an element *a* is $\frac{-a}{a+1}$.

25. Let H be the hypotenuse AC and θ be the angle between the hypotenuse and the base BC of the right angled triangle ABC.

Then $BC = base = H \cos \theta$ and $AC = Perpendicular$ $=$ H sin θ

 \Rightarrow P = Perimeter of right-angled triangle

 $= H + H \cos \theta + H \sin \theta = P$

For maximum or minimum of perimeter, $\frac{dP}{d\theta} = 0$ P_{α} dP ² $\frac{d\mathbf{d}}{d\theta}$ = 0 θ

$$
\Rightarrow H (0 - \sin \theta + \cos \theta) = 0, \text{ i.e. } \theta = \frac{\pi}{4}
$$

Now

the inverse of an element *a* is
$$
\frac{-a}{a+1}
$$
.
\nLet H be the hypotenuse AC and θ be the angle
\nbetween the hypotenuse and the base BC of the
\nright angled triangle ABC.
\nThen BC = base = H cos θ and AC = Perpendicular
\n= H sin θ
\n $\Rightarrow P = Perimeter$ of right-angled triangle
\n $\Rightarrow H + H cos \theta + H sin \theta = P$
\nFor maximum or minimum of perimeter, $\frac{dP}{d\theta} = 0$
\n $\Rightarrow H (0 - sin \theta + cos \theta) = 0$, i.e. $\theta = \frac{\pi}{4}$
\n $\left(\frac{d^2P}{d\theta^2}\right) = -H cos \theta - H sin \theta$
\n $\Rightarrow \frac{d^2P}{d\theta^2}$ at $\theta = \frac{\pi}{4} = -H\left[\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\right] = \sqrt{2}H < 0$
\nThus P is maximum at $\theta = \frac{\pi}{4}$.

4 π Thus P is maximum at $\theta = \frac{\pi}{4}$.

 $1 \leq \mathcal{N}$ and $1 \leq \mathcal{N}$ $1\frac{1}{2}$ 2

0

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\nFor
$$
\theta = \frac{\pi}{4}
$$
, Base=H $\cos\left(\frac{\pi}{4}\right) = \frac{H}{\sqrt{2}}$ and Perpendicular = $\frac{H}{\sqrt{2}}$

\nHence, the perimeter of a right-angled triangle is maximum when the triangle is isosceles.

\n $\frac{1}{2}$

Finding the point of interection of given lines as $A(1,2)$, $B(4,3)$ and $C(2,0)$ 1

Therefore, required Area

26.

$$
= \int_{1}^{4} \left(\frac{x+5}{3}\right) dx - \int_{1}^{2} (4-2x) dx - \int_{2}^{4} \left(\frac{3x-6}{2}\right) dx
$$

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\n
$$
= \frac{1}{3} \left[\frac{x^3}{2} + 5x \right]_1^1 - (4x - x^2) \Big]_1^2 - \left[\frac{3}{4}x^2 - 3x \right]_2^4
$$
\n
$$
= \frac{1}{3} \left[\left[\frac{16}{2} + 20 \right] - \left[\frac{1}{2} + 5 \right] \right] - [(8 - 4) - (4 - 1)] - [(12 - 12) - (3 - 6)]
$$
\n
$$
= \frac{1}{3} \times \frac{45}{2} - 1 - 3 = \frac{7}{2} \text{ sq. units}
$$
\nOR
\n
$$
I = \int_1^1 (2x^2 - x) dx = \int_1^1 f(x) dx
$$
\n
$$
= \lim_{h \to 0} [f(1) + f(1 + h) + f(1 + 2h) + \dots + f(1 + (n - 1)h)] - \dots
$$
\nwhere $h = \frac{4 - 1}{n}$, i.e., ph = 3
\nNow, $f(1 + \overline{n} - 1h) = 2(1 + (n - 1)h)^2$ $(1 + (n - 1)h)$
\n
$$
= 2[1 + (n - 1)^2 h^2 + 2(n - 1)h] - 1 (1 + (n - 1)h) = 2(n - 1)^2 h^2 + 3(n - 1)h + 1
$$
\nTherefore, $f(1) = 2.0^2 h^2 + 3.0 h + 1$, $f(1 + h) = 2.1^2 h^2 + 3.1 h + 1$
\n $f(1 + 2h) = 2.2^2 h^2 + 3.2 h + 1$, $f(1 + (n - 1)h) = 2.2^2 h^2 + 3.2 h + 1$
\n
$$
I = \frac{1}{2}
$$

$$
=2\left(1+(n-1)^2h^2+2(n-1)h\right)-1(1+(n-1)h) = 2(n-1)^2h^2+3(n-1)h+1
$$

$$
f(1+2h)=2.2^2 h^2+3.2.h+1
$$
, $f(1+(n-1)h)=2.2^2 h^2+3.2.h+1$ $1\frac{1}{2}$

()() ()() 2 0 –1 2 –1 3 –1 – Thus, I lim 2 *^h* 6 2 *n n n n n nh h h n h* → = + + ()()() ()() 0 2 – 2 – 3 – lim *^h* 6 2 *nh nh h nh h nh nh h hn* [→] = + ⁺ ² ()()() () 0 2 3 3 – 6 – 3 3 (3 –) lim 3 *^h* 6 2 *h h n* → = + ⁺ ⁼ ⁶⁹ 2 1 1 2 **27.** () – 2 – 3 – 7 3 –1 –1 *x y z* = = =λ ¹ (3 2, – 3, 7 λ + λ + − λ +) ¹ Since B = λ + λ + − λ + (3 2, – 3, 7) lies on 3*x* – *y* – *z* = 7 , we have = λ + λ + λ + = ⇒ λ = 3 3 2 – – 3 – – 7 7 1 () () ()

The equation of line AB perpendicular to the given plane is

$$
\frac{x-2}{3} = \frac{y-3}{-1} = \frac{z-7}{-1} = \lambda \text{ (say)}
$$
 $1\frac{1}{2}$

Therefore coordinates of the foot B of perpendicular drawn from A on the plane $3x - y - z = 7$ will be

$$
(3\lambda+2,-\lambda+3,-\lambda+7) \qquad \qquad 1\frac{1}{2}
$$

$$
=3(3\lambda+2)-(-\lambda+3)-(-\lambda+7)=7\Rightarrow \lambda=1
$$

Thus
$$
B = (5, 2, 6)
$$
 and distance $AB =$ (length of perpendicular) is

$$
\sqrt{(2-5)^2 + (3-2)^2 + (7-6)^2} = \sqrt{11}
$$
 units

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uus B = (5, 2, 6) and distance AB = (length of perpendicular) is
 $(2-5)^2 + (3-2)^2 + (7-6)^2 = \sqrt{11}$ units

ence the co-ordinates of the foot of perpendicular is (5, 2, 6) and the length

rpendicular = $\sqrt{11}$

O Hence the co-ordinates of the foot of perpendicular is (5, 2, 6) and the length of perpendicular = $\sqrt{11}$ 1 EXECTIVE MATHEMATICS
 $B = (5, 2, 6)$ and distance AB = (length of perpendicular) is
 $2-5)^2 + (3-2)^2 + (7-6)^2 = \sqrt{11}$ units

circle the co-ordinates of the foot of perpendicular is (5, 2, 6) and the length of

pendicular = \sqrt

OR

$$
\vec{r} = \hat{i} + \hat{j} + \lambda \left(\hat{i} + 2\hat{j} - \hat{k} \right)
$$
 -------(*i*)

and
$$
\vec{r} = \hat{i} + \hat{j} + \mu \left(-\hat{i} + \hat{j} - 2\hat{k} \right)
$$
 -----(ii)

and has $d.r.'s$, -1 , $1, -2$

Since the required plane contain the lines (i) and (ii), the plane is parallel to the vectors

$$
\vec{b} = \hat{i} + 2\hat{j} - \hat{k}
$$
 and $\vec{c} = -\hat{i} + \hat{j} - 2\hat{k}$

$$
1 \\
$$

$$
\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = -3\hat{i} + 3\hat{j} + 3\hat{k}
$$

1

Hence equation of required plane is

() () *r a b c* – . 0 × = *r i j i j k* – . 3 3 3 0 () () ⇒ + − + + = ⇒ + + = *r i j k* . – 0 () () 2 2 2 1(–1) 1.1 1.1 1 1 1 + +− + +

$$
\Rightarrow \vec{r} \cdot (\vec{-} \vec{i} + \vec{j} + \vec{k}) = 0
$$

and its cartesian form is $-x + y + z = 0$

Distance from $(1, 1, 1)$ to the plane is

$$
\frac{|1(-1)+1.1+1.1|}{\sqrt{(-1)^2+1^2+1^2}} = \frac{1}{\sqrt{3}} \text{unit}
$$

28. Let *x* denote the number of kings in a draw of two cards. Note that *x* is a random variable which can take the values 0, 1, 2. Now

IDENTIFY: 133

\nHence equation of required plane is

\n
$$
\left(\vec{r} - \vec{a}\right) \cdot \left(\vec{b} \times \vec{c}\right) = 0
$$
\n
$$
\Rightarrow \left[\vec{r} - \left(\hat{i} + \hat{j}\right)\right] \cdot \left(-\vec{3}\hat{i} + 3\hat{j} + 3\hat{k}\right) = 0
$$
\n
$$
\Rightarrow \left[\vec{r} - \left(\hat{i} + \hat{j}\right)\right] \cdot \left(-\vec{3}\hat{i} + 3\hat{j} + 3\hat{k}\right) = 0
$$
\nand its cartesian form is - x + y + z = 0

\nDistance from (1, 1, 1) to the plane is

\n
$$
\frac{|1(-1) + 1 \cdot 1 + 1 \cdot 1|}{\sqrt{(-1)^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}} \text{ unit}
$$
\nLet x denote the number of kings in a draw of two cards. Note that x is a random variable which can take the values 0, 1, 2. Now

\n
$$
P(x=0) = P(\text{noking}) = \frac{48!}{52 \text{ c}_2} = \frac{2!(48-2)!}{52!} = \frac{48 \times 47}{52 \times 51} = \frac{188}{221}
$$
\nP(x=1) = P(\text{one king and one non-king})

\n
$$
= \frac{^{4}C_{1} \times 48C_{1}}{^{5}C_{2}} = \frac{4 \times 48 \times 2}{52 \times 51} = \frac{32}{221}
$$
\nand P (x = 2) = P (two kings) = $\frac{^{4}C_{2}}{^{5}C_{2}} = \frac{4 \times 3}{52 \times 51} = \frac{1}{221}$

\nThus, the probability distribution of x is

 $P(x = 1) = P$ (one king and one non-king)

$$
=\frac{{}^{4}C_{1} \times 48C_{1}}{{}^{52}C_{2}}=\frac{4 \times 48 \times 2}{52 \times 51}=\frac{32}{221}
$$

and P (x = 2) = P (two kings) =
$$
\frac{{}^{4}C_{2}}{{}^{52}C_{2}} = \frac{4 \times 3}{52 \times 51} = \frac{1}{221}
$$

Thus, the probability distribution of *x* is

Now mean of $x=$ E 1 *n i*=1

$$
= 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + \frac{2 \times 1}{221} = \frac{34}{221}
$$

$$
E(x^2) = \sum_{i=1}^n x^{i^2} p(xi)
$$

$$
=0^{2} \times \frac{188}{221} + 1^{2} \times \frac{32}{221} + 2^{2} \times \frac{1}{221} = \frac{36}{221}
$$

 Now

rics
\n
$$
\frac{88}{21} = \frac{32}{221} = \frac{1}{221}
$$
\n
$$
= E(x) = \sum_{i=1}^{n} x_i P(x_i)
$$
\n
$$
= 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + \frac{2 \times 1}{221} = \frac{34}{221}
$$
\n
$$
E(x^2) = \sum_{i=1}^{n} x_i^2 P(x)
$$
\n
$$
= 0^2 \times \frac{188}{221} + 1^2 \times \frac{32}{221} + 2^2 \times \frac{1}{221} = \frac{36}{221}
$$
\n
$$
Var(x) = E(x^2) - [E(x)^2] = \frac{36}{221} - \left(\frac{34}{221}\right)^2 = \frac{6800}{(221)^2}
$$
\nand
\nand
\n
$$
Var(x)
$$

MATHEMATICS
\n
$$
\frac{x}{P(x)} = \frac{0}{188} = \frac{1}{32} = \frac{1}{221}
$$
\n
$$
= 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + \frac{2 \times 1}{221} = \frac{34}{221}
$$
\n
$$
= 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + \frac{2 \times 1}{221} = \frac{34}{221}
$$
\n
$$
E(x^2) = \sum_{i=1}^{n} x^2 p(x)
$$
\n
$$
= 0^2 \times \frac{188}{221} + 1^2 \times \frac{32}{221} + 2^2 \times \frac{1}{221} = \frac{36}{221}
$$
\nNow $\text{var}(x) = E(x^3) - [E(x)^2] = \frac{36}{221} \left(\frac{34}{221}\right)^2 = \frac{6800}{(221)^2}$
\nTherefore standard deviation $\sqrt{\text{var}(x)}$
\n
$$
= \frac{\sqrt{6800}}{221} = 0.37
$$

\nLet the mixture contains *x* kg of food I and *y* kg of food II.
\nThus we have to minimise

29. Let the mixture contains *x* kg of food I and *y* kg of food II.

Thus we have to minimise

$$
Z = 50x + 70y
$$

Subject to

$$
2x + y \ge 8
$$

$$
x + 2y \ge 10
$$

$$
x, y \ge 0
$$

1

The feasible region determined by the above inequalities is an unbounded region. Vertices of feasible region are

A (0, 8) B (2, 4) C(10, 0)
$$
\frac{1}{2}
$$

$$
B(2, 4)=380 \quad C(10, 0)=500
$$

As the feasible region is unbounded therefore, we have to draw the graph of

$$
x + 70y < 380 \text{ i.e. } 5x + 7y < 38
$$
 $\frac{1}{2}$

As the resulting open half plane has no common point with feasible region thus the minimum value of $z = 380$ at B (2, 4). Hence, the optimal mixing strategy for the dietician would be to mix 2 kg of food I and 4 kg of food II to get the minimum cost of the mixture i.e Rs 380.